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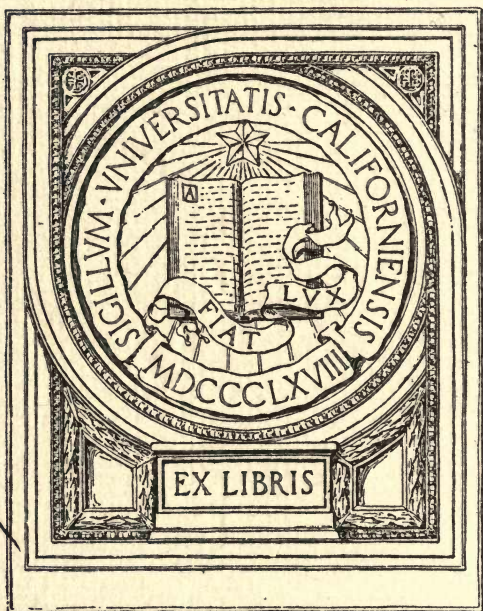
PREPARATION
OF THE CHILD
FOR SCIENCE.
By M. E. BOOLE

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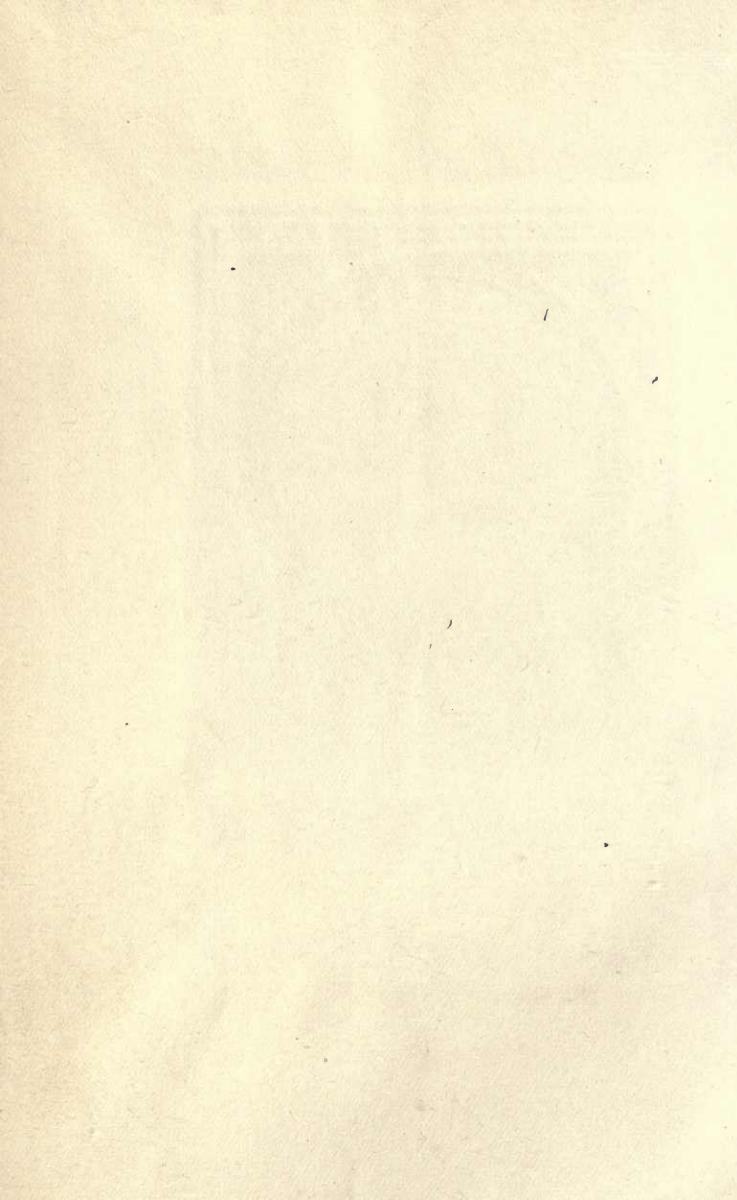
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THE
EXPLANATION OF THE
FIELD FOR SCIENCE

BY M. E. BOOSE

WITH A PREFACE BY THE AUTHOR

OXFORD
AT THE CLARENDON PRESS



THE PREPARATION OF THE CHILD FOR SCIENCE

By M. E. BOOLE

AUTHOR OF 'LECTURES ON THE LOGIC OF ARITHMETIC'



OXFORD
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TO
ETHEL GERTRUDE EVEREST

MY DEAR COUSIN,

Three-quarters of a century ago your father, during a visit to his native land, infused into the minds of a few young mathematicians, among whom were Charles Babbage and J. Herschell, certain ideas about the nature of man's relation to Unknown Truth which underlay all science in ancient Asia, and which he had learned from Brahman teachers. The seed which he then sowed has borne abundant fruit in English Mathematics. Of his subsequent work in India some have sought to express their appreciation by giving his name to a great inaccessible snow-peak. You and I think that we shall more truly fulfil his ambitions by making as accessible as we can to little children in all parts of the Empire that open gateway to the Unseen at which he stood in perpetual adoration to the last hours of his life.

M. E. BOOLE.



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PREFACE

THE following volume is an attempt to utilize on behalf of little children the life-work of many great men now almost forgotten. Amongst these may be specially mentioned :—

NICOLAS ANTOINE BOULANGER. He left school a reputed dunce, incapable of learning algebra. He died in 1759, in his fortieth year, a self-taught mathematician and an eminent engineer. He left behind him in MS. two works, *L'Antiquité dévoilée par ses usages* and *L'Origine du despotisme oriental*, in which he explained the connexion of fixed mental attitude with the enslavement of the masses, and of periodic reversal of attitude with mental emancipation and power. His books were published after his death in several editions, but seem to have mysteriously disappeared soon afterwards.

THOMAS WEDGWOOD. He was assisted in his scientific researches by Erasmus Darwin (grandfather of Charles Darwin), a then famous physician. His life-long dream was to promote happiness on earth, not by the multiplication of

mechanical appliances for comfort and pleasure, but by the evolution of a Race gifted with powers of intellectual enjoyment, larger than those of man as he now exists. He made a special study of the faculty which he called Genius and the conditions of its orderly development. By Genius he evidently meant, chiefly, the power of seeing truth at first hand. The MS. of his work was entrusted to a friend who is said to have lost it. The rough notes for the book in Wedgwood's handwriting, with a fair copy of one or two of the chapters, were found in 1883. Among the papers was a scrap on which was written, in a shaking hand, the sentence:—'How exhilarating is the thought that if, by the labour of my whole life, I can add one idea to the stock of those concerning education, my life has been well spent.'

CHARLES BABBAGE. By comparing European mathematical processes with certain ideas about the development of human faculty derived from Asiatic sages, he was able to realize the nature of some thought-processes so vividly as to conceive, in imagination, what may almost be called an Automatic Thinker of iron and brass. His calculating machine, if completed, would sometimes have rung a warning bell and put up a signal:—'More data wanted;' and oc-

casionaly it would have put up the signal:—
'Data incompatible with each other.' The machine was, however, not completed ; it is kept in the Mathematical Department of the South Kensington Museum. Babbage's book and pamphlets, more valuable than the machine would ever have been, are practically unknown to the majority of the scientific world.

GEORGE BOOLE. My husband carried out Babbage's idea by means, not of a machine, but of a self-acting algebraic notation, which automatically produces the symbol for 'insufficient data' ($\frac{0}{0}$), and that for 'these conditions are incompatible in the sphere of human consciousness' ($\frac{1}{0}$). He also carried out into other automatic notations Babbage's suggestions about the psychological value of certain traditional opinions on metaphysical subjects. The notations adopted by him have been used in the application of mathematics to the Physical Sciences ; the use of them for this purpose would be largely facilitated if mathematical teachers had any clear idea of their meaning and origin. His books are still used in Germany and Austro-Hungary for educational purposes. In England they have been superseded by others in which the same notations are employed, but in a manner which tends to conceal their

history and genesis. George Boole was abundantly rewarded in his lifetime with medals and honorary degrees ; but his books are now out of print.

RAM CHUNDRA. Treatise on Maxima and Minima, with introduction by J. Herschell and A. De Morgan about ancient Asiatic Methods.

GRATRY : A French Oratorian. His work on Logic contains a valuable analysis of the psychology of the Infinitesimal Calculus ; which he, as did afterwards Hinton, treated as proof of the importance to intellectual progress of rhythmic alternation between the study of detail and the synthesizing of details. He asserts that Geometric Induction is a process of prayer ; by which he means an appeal from the finite to the Infinite for light upon finite concerns. The mathematical portion of his work is out of print.

BENJAMIN BETTS, formerly of the Government Survey of New Zealand, after much study of Fichte's philosophy, invented a peculiar system of co-ordinates, by means of which he illustrated diagrammatically the expansion of thought under pressure from opposing forces. To his surprise his diagrams turned out to resemble the outlines of many known leaf and flower forms,

as well as of some not known on this planet. He came to conclusions on vegetable morphology in some respects similar to those arrived at quite independently by James Hinton and expressed in his 'Life in Nature.'

Many years ago I enjoyed the privilege of reading Mr. Betts's MS., and had a long correspondence with him. His system of coordinates cannot be much used except by those whose hands have been trained in the work of the mechanical draughtsman. It has however been very helpful in suggesting simpler methods. The following quotation from a letter of Mr. Betts will give some idea of the nature of his philosophy. 'Suppose yourself placed at an almost infinite distance on the planetary axis of our solar system, and, focussing (in imagination) aeons of time into a few seconds, and millions of miles into the optical angle, view the condition of our solar system in its passage in the direction of Hercules. What would be visible? . . . The laws of infinite solar systems throughout the universe are telegraphed to us by the flowers at our feet. . . . These laws of the corolliform we have educed as Laws of Thought, of self-consciousness as revealed in the act of thinking. . . . Some one is thinking there, thinking in potential activity, as you and I are thinking this moment.'

JAMES HINTON analysed carefully the Psychology of the processes for the Summation of Series and for solving Quadratic Equations ; as well as that of the Fluxion method.

DR. CHARLES WINSLOW instituted a series of investigations on Force-lines as co-ordinates for illustration of Laws of Thought. At the same time he corresponded with ladies on Mission Stations and other persons residing in Volcanic districts, who, during many years, sent to him observations on minute points connected with seismic disturbance. After a life of heroic struggle against many difficulties, he published a book, *Force and Nature*, in which he advanced a psychologic theory connected with Geologic Evolution, which at some points coincided with the theories of J. Hinton ; at some with that of Betts ; at others, again, it reminded me of my husband's system of psychologic co-ordination. He and James Hinton once compared notes in my presence.

The late DR. WILTSHIRE was well known as a physician. It was not so well known that he had made a profound study of the effect on the nerves and brain of periodic alternation of mental attitude ; not mere repose, but reversal of attitude as preparation for repose. He believed

that the habit of this is conducive, not only to longevity, but to that prolonged youth of the mind which makes old age profitable. He had come to the conclusion that most of the revealing power born into the world is wasted for the service of mankind, by too early precipitating itself in some concrete form, or committing itself to some definite view or opinion. He said:—‘The way to do good work is to live to be old. If you have genius, keep it fresh till you have experience as well.’ He expressed to me certain conclusions to which he had come about the bad consequences, to the general morality and health, of the people being educated in habits of slavery to fixed mental attitude.

GRATRY held that the Art of Thinking would never flourish during periods of intellectual individualism and competition; that the essential condition for any marked evolution of it is one of self-effacing communal study. This condition of generous selflessness prevailed in Mathematical Science during the great revival in which A. De Morgan and my husband took part, in the forties and fifties of the last century.

For the last thirty years there has existed an informal group of workers, organized on the communal plan suggested by Gratry, which

has for one of its main objects the study of the Laws of Sequence of Thought-processes. Among its rules are the following :—

Any member is at liberty to communicate to the public any information on psychology or education which he acquires by the assistance of other members.

Any book or article emanating from the group is to be signed by the member who is responsible for the selection and arrangement of the contents. He owes to his colleagues no public recognition, and is expected not to connect their names with any views which he may put forward.

It only remains for me therefore to express my warm thanks to Miss Eleanor Cobham, whose intelligent and devoted assistance has made it possible for me to organize into a form available for publication the mass of material relating to the Psychology of the process of Investigation which has drifted into my hands in the course of a long life.

MARY EVEREST BOOLE.

CHAPTER I

THE SCIENTIFIC MIND

THE present volume is specifically concerned with the preparation of the child for science. It offers suggestions as to means by which the scientific condition of mind can be induced. Each mother must decide for herself whether, and to what extent, she wishes to induce this condition. The first desideratum is that parents should form a clear idea what is the scientific condition of mind; the next is that they should know in what consists the preparation for it.

The typically scientific mind may be described as one which stands in a definite relation to As-Yet-Unknown Truth, and especially to that portion of the As-Yet-Unknown which is just below the horizon of knowledge. In proportion as a mind is non-scientific, the occurrence of an unfamiliar phenomenon stimulates it to form some immediate classification or judgement. A new statement is hailed at once as 'true' or 'false'; a new fact is classified as 'good' or 'bad,' 'nice' or 'nasty'; an unfamiliar action

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as 'right' or 'wrong,' &c. In proportion as a mind is scientific, the occurrence of a new phenomenon tends to set it vibrating with a consciousness of coming revelation, and to start a certain cycle of mental attitudes, a cycle of the following kind :—

Homage.

Attention.

Observation.

Analysis.

Antithesis.

Synthesis.

Contemplation.

Effacement.

Repose.

Judgement or Classification.

The cycle varies in duration ; each phase may occupy a few seconds, or many months, or even years ; but the tendency to fall into some such sequence as that above described at the touch of a new fact is what constitutes the essentially scientific condition.

It would, of course, be impossible to direct the studies of a child strictly in accordance with any such sequence as the above, but a good deal may be done towards inducing scientific elasticity by setting up the habit of rhythmic alternation of attitude. And, if the

child is to be prepared for science, something of reverence should always accompany the impact of the As-Yet-Unknown.

That delicate sensitiveness to the touch of the illogical, to the limits of knowledge, and to the Presence of the As-Yet-Unknown, which it was the object of great mathematicians to confer on automatic mechanism, is too often destroyed in the human brain by rough and ready processes, adopted, sometimes for the purpose of fixing the opinions of young people, sometimes for that of enabling them to pass examinations successfully in subjects which they do not really understand. To cultivate it in the young child is the object of some of the precautions recommended in the following pages. None of them, however, are intended to be slavishly carried out.

Many thoughtful parents are beginning to wish that their children, while learning the elements of sciences, could also gain an early initiation into those methods of study by following which the Newton, or Darwin, or Faraday of the future trains himself after his teachers have done with him. The methods in use in schools ought, they think, to differ from the Newton, Darwin, Faraday methods of private study only as the little sapling, which still needs support and shelter, differs from the tree.

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It seems a not unreasonable desire that the sapling of early method should have some natural tendency towards growing into that particular tree, true science-method. Whereas, so far as I can learn, most of the recognized school methods differ from the essential science-method rather as the little rose bush or cabbage differs from the oak: the cabbages and rose bushes may bring forth results admirable, charming, and vigorous, which will be good to eat or pleasant to the eye, and productive of prizes at shows; but they have no sort of tendency to grow into an oak; and if we happen to want to plant an oak-tree in the place where they have been set, we shall have to grub up some of the roses and cabbages. The youth who means to make scientific philosophy his career usually has to begin by grubbing out of himself what he learned at school; not of course all the statements about fact, and still less the good habits of order and industry, but all that he learned at school in the way of *method of study*. There are schools to which this sweeping criticism does not apply; here and there a teacher of elementary science evolves unconsciously, by some accident of organization or instinct, the true secret of the Newtons and Darwins. The pupils of such teachers are not always particularly successful in their early

studies, for a reason which I will explain further on; the sapling does not grow fast, owing to uncongenial surroundings; but it is of the true breed, and, though a starveling, sound at the core. But scientific method is something more definite than the occasional and unconscious action of a heaven-born teacher; it might be, and ought to be, explained by lecturers on pedagogy to audiences of intelligent pupil-teachers.

There is a principle which has been formulated in scientific treatises on the Art of Study, such as that of Gratry, and which is so recognized in the circles where science is being evolved as to be held a matter of course not needing to be stated at all, unless when some science leader is roused to protest by the proceedings of those around him. An instance of such protest may as well be given now. Forty years ago a lady asked me to lend her for her children some good introduction to the study of nature. I lent her a work which had been the delight of my own childhood; but she promptly returned it with a note to the effect that she could not allow her children to read it, for it was antiquated and not up to modern requirements; it stated that herring shoals approach Europe from the North, whereas it had lately been discovered that they come from the South (or vice versa—I do not at

this moment remember which theory was held forty years ago and which twenty years before that); and, she added, there is so much that children must learn nowadays that they must not waste their time in learning theories already exploded. A scientific man to whom I showed the letter remarked impatiently that the writer evidently had no idea what she was about; for, he said, what is most necessary for the children to learn is not what is the last new theory about where herrings are hatched, but how to extract the truth from a series of impressions and statements, each of which is only partially true. In this utterance we have, I think, the key to the most essential element in a truly scientific method of study; and the case is worth analysing, for it illustrates the grave error, the hidden pitfall, into which the advanced section of the educational profession fell in passing from the ancient ideal of an education based on classics to the modern ideal of an education based on science.

Dead languages are fixed for ever, at least as to those parts of the study with which young people are concerned. The dative of such a noun is so-and-so; this particular word is a dactyl and that a spondee; and the duty of the schoolmaster was to see that the boys spelled and scanned the words accordingly.

My friend had been trained in the fundamental conception that there is a right way of spelling a word, and other ways are wrong ; and that children should be taught the right way. She had so far outgrown the old classic traditions as to think that it is not enough to know how Virgil scanned Latin, and Aeschylus wrote Greek ; that a citizen of the nineteenth century should know how God makes the world and whence comes humanity's food ; she had read enough science, too, to be aware that it is a growing, not a fixed and limited form of knowledge. She was not foolish enough to imagine she could get hold of any absolute and final opinions about science, to substitute for the absolute and final right ways of writing Latin ; so she enshrined instead, on her domestic altar, the last new up-to-date opinion, having no conception of the true nature of eternal scientific method. What this lady did in detail the advanced section of educationalists have done as a whole ; when they discovered that fixed rules of grammar are not, as used to be thought, sufficient mind-food, they substituted, not the eternal truth of mental pulsation, but up-to-dateness. The newest textbooks, the latest opinion about evolution, the most newly-invented and costliest apparatus, the latest method of showing electric or galvanic action—

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these were supposed to be the characteristics of the best science school ! Yet it is well known that many of our great scientific leaders learned at first from books old even in their childhood, and used simple apparatus of their own contriving for lack of means to purchase any other.

In classical learning it is eminently desirable to secure that the right impression shall be made from the first ; that wrong impressions shall have as little time as possible to deepen themselves. We wish the child's eye and ear to become accustomed from the first to welcome the right and reject the wrong ; we do not wish him to gain any habit of tolerating wrong impressions. If the child uses the nominative where he should use the accusative, and is not at once corrected, that is so much to the bad for his future progress ; if he can be got not to be able to remember a time when he used the word wrongly, that is so much to the good. But in science there are, there can be, no absolutely right impressions ; our minds are not big enough to grasp any natural fact as a whole ; everything depends upon drawing right conclusions from combinations of impressions, each of which is in itself inadequate and partially misleading ; and if the pupil is to be got into scientific methods, that is what he must be trained to do. And in order that he may learn

to do it, it is sometimes necessary that each of a succession of 'wrong' impressions should have time to register itself on the brain and become part of its available stock. Such a statement may naturally convey to the scholastic mind trained in classic traditions an impression of disorderliness, but it does not imply disorder. Up-to-dateness is the cause of disorder; the haste, the greed, to efface rapidly each partial impression, when we have nothing to substitute for it but some other impression equally partial, is not only unscientific but eminently disorderly.

Some readers may be tempted to think that in mathematics there are no tolerated and mutually corrective errors. The space at my disposal does not admit of entering fully into that question: it is enough to say here that mathematical method is the typical and perfect scientific method, in which the warp and woof of *polar partial apprehensions* are so fine and so perfectly woven that the texture is, so to speak, hardly visible to the naked eye; but it reveals itself on close analysis of mathematical procedure; and, indeed, it was by means of mathematical investigation that the principle underlying all true scientific mental action was brought to formulation.

The difference between teaching sciences and training in scientific method is well illustrated

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in the teaching of botany and biology by dissecting flowers and animals. In the first place, as Professor Geddes constantly reminds his hearers, confusion of mind is induced by using the word 'biology' to anything which involves destroying the life of the object under investigation. For purposes of scientific investigation, destruction is often necessary (though there need not be nearly as much of it as there is). But, however necessary, it is in itself not biology but necrology, for it shows the pupil not how the thing lives, but how it looks when dying or dead. And this is more than a mere quibble about words ; the whole question of scientific method is involved. Children inherit from their monkey-ancestors an abundant supply of inquisitive destructiveness ; the quality which needs *educating*, i.e. *educing*, is reverent patience to stand still and watch other creatures living their lives. Science has been evolved by the balanced action of these two faculties or powers—the natural monkey-inquisitiveness and the educated spiritual reverence. To over-cultivate destructive curiosity is a cheap way of enabling children to make a show in science examinations ; but that sort of thing has no tendency to induce the habit of true scientific method, which depends on alternation of opposites. If a child is to be

initiated into scientific method, its early studies must go in the direction of *balancing* its natural destructiveness by learning to foster life and to study patiently the development of living things. Of course, this ideal is very difficult to carry on in town ; it is, for instance, difficult to study botany in town except by means of cut flowers ; but a good deal might be done by teaching children, when they see a flower, either growing or in water, not to touch it till they have learned all they can of its poise and mode of growth, so as to be able, after dissecting it, to reconstruct in their minds an accurate picture of how it looked before they disturbed it.

The tendency in science classes to an over-cultivation of mere monkey-destructiveness has been largely fostered by the claim of certain faddists that children shall be told nothing, but shall 'find out everything for themselves.' What science really does claim in this matter is that a clear line of demarcation shall be preserved between *what the individual has observed* and *what he has learned at second hand*. The claim that children shall find out everything and be told nothing is palpably absurd.

As a matter of fact, most science leaders read eagerly and omnivorously on their own subject, being anxious to get all the help they can from

previous and contemporary investigators. No man can find out for himself all he needs to know. How could any individual find out by himself the earth's movement round the sun? What science does claim is, that no child shall be told anything about the motion of the earth till he has observed many sunrises and sunsets; till a clear sense-impression of the earth standing still and the sun moving has become organic within him. This registering of a 'wrong' impression is what in science we have to secure; while in classics we should try to prevent it.

Now let us see how a true science teacher deals in detail with this scientific necessity for registering, separately, successive partial and misleading impressions. Her subject is, we will say, the buttercup. First, she gets the children to look carefully at the plant as a whole; if possible, to watch it growing. If they can have a specimen in their garden, and watch the gradual unfolding of leaf and flower-buds, so much the better. If she can draw, she will teach them to register with the paint-brush their impressions of it at various stages. She will lead them to observe, among other facts, that it differs from such flowers as the larkspur by having apparently no nectary. When they have carefully noticed all she can get them to see of the living plant, she proceeds to a

dissection of the flower. 'What is this?' she says, pointing to the scale at the base of a petal. 'A bit of petal turned up,' says one child. 'A stamen with the anther knocked off,' says another. 'I think it looks like a little petal, like the inside petals of a double anemone,' says some specially observant little person. If the teacher is a mere crammer, she will immediately explain the function of the scale; but if she is a real science teacher, she makes no comment; she only asks questions to draw out further observations.

Some other day she will explain that the scale is believed to secrete some substance, and, therefore, to partake of the nature of a nectary. But, if she is wise, she will not do this till the conception of an anomalous organ, as to which one could not be sure whether it is a petal or a stamen, has had time to soak itself well in. Then, when the children learn that it is, after all, a sort of nectary, though it is not the shape of one, only that it is not called nectary because the substance it secretes is not honey, this will be a real introduction for them to the conception of metamorphism. It will not enable them to answer questions at an examination on Goethe's theory of metamorphism, still less on any later one; but it will prepare them to understand metamorphism as Goethe understood it, by

leading them to see nature as Goethe saw it. Meantime, if the up-to-date mother or head mistress has been present at the former part of the lesson, this is the kind of comment she will probably make:—‘Miss Dash came to us with the character of a high-class science teacher ; but I am disappointed in her ; she is so dull and behindhand ; she constantly lets mistakes pass. Only to-day I heard Bessie call the little scale of the buttercup a bit of petal ; and Miss Dash seemed not to know any better ; at any rate she said nothing. I am sure I read in the Botanists’ Magazine that the scale is considered to be a secreting gland. It is very provoking, after all the pains I took to secure good teaching in that department.’

Such is the kind of misapprehension which has come of trying to put the new wine of scientific progress into the old bottle of academic rightness. A mistake in grammar is a mere blunder, and should be corrected as soon as possible, so as to prevent the child’s eye and ear from becoming accustomed to, and contented with, inelegant diction. But the misapprehension about the gland is not a blunder ; it is the normal impression which the phenomena should naturally make on a mind at a certain stage of development ; and it is the teacher’s business to ensure that her pupil’s mind shall register that

impression before being disturbed by the intrusion of one derived from more recondite investigations. If, indeed, the child were, from carelessness, to make what is for him, at his present stage, a blunder, she would immediately lead him by questions to see his error ; but that is quite another matter. In arithmetic—the most methodized of sciences—we make the child correct his sum if he has made a careless mistake ; but no good teacher interrupts a partial apprehension till the normal time has come. Scientific education is not arrived at, and never can be arrived at, by young people being crammed at School and College with ready-made knowledge ; and left to find out, after adult life and its duties have come on them, that they are still very ignorant of how to learn what they now need to know. Nor do they mend the matter then, by turning in disgusted recoil from the sham knowledge with which others crammed them, to cram themselves, from books, with ready-made Theories about the limits of the knowable and the ‘Philosophy of the Unknowable.’ Scientific culture is the result of a steady life-long habit of friendly and intimate, though reverent, intercourse with the Eternally Infinite Unknown.

We must now go a little further into the

questions:—What is that essence of scientific method on which its essential scientific quality depends? What are the conditions for securing it? How has it been introduced into science colleges? How might it be introduced into elementary teaching? Are there any special difficulties in so introducing it; and, if so, how can they best be overcome?

An individual gets his information about any given class of facts from a variety of sources: his own observations on facts; what he learns on more or less reliable testimony as to what others have observed; theories, some true, some false, which have been held by human beings to account for the facts; and his own thinking about the facts. The essence of scientific method, so far as I at all understand the great men who have formulated it, consists in keeping the produce of the three external sources of information *apart in the mind* till the lid is shut down, so to speak, on the outer world, and the process of thinking has begun.

A man is not reliable as a discoverer in science unless he can say, 'Such-and-such a fact I myself observed; such another I read about and afterwards verified by my own observation; such-and-such statements I have read, but have not yet verified; such another I have read, but my own observation points to the contrary

opinion.' This is not a mere question of priority, of who deserves the honour of a discovery : we do not trust the *scientific qualities* of one who does not *feel* quite differently about a fact which he himself observed from the way he feels about something he has read or heard¹. And even if he is making this distinction, it does not follow that he has performed any scientific act of mind. Not till he has thought in silence, not till different kinds of knowledge derived from various sources combine to form a mental impression, can true scientific action be said to have taken place.

Suppose a youth gazes at the starry heavens till he has soaked in an impression of their varying aspect, that is instructive ; but the instruction is not scientific. Suppose a navigator reads or is told that when the heavenly bodies appear in certain relative positions at a certain hour the latitude must be so-and-so, that is useful technological instruction : it has no claim to be called scientific. (Technologic information is often miscalled 'scientific' in advertisements.) Suppose we read up a history of the various theories which have been held as to the causes of phenomena presented by the heavenly bodies, that is in itself a historic or literary treatment of the subject, not a scientific

¹ See pp. 143 et seq.

one. But when a child has formed for himself a clear, undisturbed impression of the earth's unmovableness and the apparent motion of the sun, and then has read that astronomers believe the earth goes round and the sun does not go round the earth, if he then puts together in his mind the two apparently conflicting statements—that made by his senses, and that made by his book—and lets them combine to create in him an impression which shall embrace both, then the sacrosanct scientific act has taken place within his mind. He has really done a bit of true science work.

In science colleges this *sine qua non* of scientific method is to some extent provided for. In the laboratory the youth is helped to observe for himself. In private study he has to read textbooks, in which are recorded the observations made by others; he attends lectures, in which is narrated the history of the successive theories by which the phenomena have been variously explained. The partial impressions are given to him separately, at different hours; usually in different rooms and by different professors. Each, therefore, makes its own distinct and separate record on the mind. And, of course, every student worth mentioning takes time occasionally to *think*, i. e. to make silence in his soul, as Gratry calls it;

to bar the access to all fresh impressions and let the impressions already made come together and correct each other. In a science college, the arrangements being such as I have described, the mere laws of collegiate good conduct generate the essential scientific method without need of any formal defining of the method itself; the rule for the pupils is to give their whole attention to what they are doing at the time; the rule for the teachers is that each is to abstain from intruding on, or interfering with, the work that is being done at the moment by any other. Each has his hour wherein to make his own impression. Each knows that the impression being made by any other is partial and incomplete, and at times even momentarily misleading; each knows that he himself could often make the immediate impression more correct by interposing; each knows that he must not so interpose till his own hour for speaking has come; he must be content to keep silence till the partial—often from his point of view partially erroneous—impression made by his fellow teacher has had time to stamp itself fully on the pupil's mind.

The organization of a science college does really correspond in its main features to what is essentially a scientific method of study.

And now we come to the questions :—What are the special difficulties in the way of introducing true scientific method into elementary teaching? And how can these difficulties best be met?

The faculties of a child are easily fatigued, and the nature of his work must be constantly varied. We cannot give him an afternoon in the laboratory, a whole hour's lecture on theory, an hour's reading up about facts, a long walk occupied in trying to think out a problem; the attitude, the mode of study, must vary every few minutes. But we cannot every few minutes change the teacher, the room, the whole *mise en scène*, as is done in college in passing from laboratory to lecture-hall; one elementary teacher must represent in turn at least three persons: the lecturer, the laboratory teacher, and the author of the textbook used for reference. Nor can the child be trusted to think in silence unhelped; the attention of children wanders and is in need of assistance from the teacher. She must be constantly on the watch to make the necessary mental silence for him, by recalling his attention from external facts and from irrelevant topics to the special point under consideration. She therefore has, as it were, not only to act the parts of two teachers and a textbook; she must also

preserve access for a fourth, that Formless One, the inner voice, with which the adult science student holds converse when he retires into silence to think out the meaning of apparently conflicting evidence. If she is really to introduce the child to scientific method, if she is to make in her schoolroom the peculiar mode of sequence on which true science-method depends; she must somehow contrive, single-handed, to keep the four departments of the study separate. And not only is there no external change of *mise en scène* to help her; there can be no conventions of external courtesy to check her; if she is tempted to let one of her functions interrupt another, no consideration of discipline or of good breeding comes to her aid in resisting the temptation. She can by no possibility conform to anything that deserves the name of scientific method except by cultivating such inner courtesy of all her various functions to each other that each will keep silence when another is making some partial impression by which the pupil is evidently partially misled.

When a really great scientific man gives an elementary lesson, he sets the example of such inner courtesy; and it is just this peculiar reticence which gives so fine a flavour of high class science to the teaching of very elementary subjects when the teacher is a very great man;

and, as I said before, one sometimes meets with a kindergarten teacher or nursery governess who adopts the great man's method by virtue of sheer heaven-sent instinct. She does truly sow the seed of true scientific method. But she sometimes has a hard time of it, and her sapling has to grow in adverse conditions, owing to misapprehension on the part of her employers.

We know from natural history what a tendency there is among weak, helpless, inefficient creatures to imitate, as a measure of self-protection, the colouring and outside appearance of the strong and efficient. Parents should remember that this is especially the case among teachers. The efficient and intelligent teacher learns all he or she can of principles of psychology and general tendencies, and then carries them out by methods which, in detail, are his or her own. It is the ignorant and inefficient teacher who makes a parade of having adopted the last up-to-date device, or the one which happens to have been mentioned in connexion with some recent success in examination or good report of an inspector.

Children must be drilled into the power of learning exactly what it is that other people have said, for those who lack this power lack the continuity which is the cause of human

progress and the condition of truly human thought. No better way has yet been found of acquiring the power of taking in what others say, than by studying the languages in which other people speak. In an admirable work addressed to parents, *Some Observations of a Foster-Parent*, W. Tarver gives excellent reasons for supposing that a thorough grounding in Latin is a better preparation for a scientific career than learning 'sciences' at school. The classical tripos man is obliged to give an account of the opinions held by ancient writers on physics, astronomy, and natural history, *opinions which neither he nor the examiner is supposed to endorse*. This is an excellent corrective of that shallow up-to-date-ness which is the vice of modern pseudo-scientism.

My advice to parents therefore would be this:—If you wish to make of your children showy pseudo-scientists, abandon all notion of true scientific method. Engage a smart up-to-date governess; who, whenever they say anything that would not bring good marks at an examination, will pounce upon it at once and insist on their saying whatever she thinks would satisfy an examiner. But if you wish them to get an insight into the way in which great science is created, give no more time to

science than you can afford to let them spend in the really scientific manner. Do not let them, while very young, prepare for any examination in natural or physical science. Give them a good foundation of training in languages and literature. For science itself get a teacher who not only loves but reverences the works of the Creator. Leave the lover of nature to teach as she is inspired by the Author of nature. And if you find that she is sometimes inspired to refrain from giving information while the child is trying to observe, or from distracting him by the intrusion of ready-made theories when he is thinking out a question for himself, be scientific enough yourself to restrain your own impatience ; do not rush in where great scientific men would be respectfully silent ; make an act of mental homage as you would at the threshold of a temple ; for the place into which you are tempted to intrude is the holy ground of science.

It would seem to be of great importance that children should learn to connect laws of thought with lines of force or motion. Every now and then some man appears in whom the instinct to express psychologic truth in terms of physical movement or of physical form is innate and overwhelming. Such a man may have all the characteristics of the great discoverer, and lack

nothing of what constitutes the great scientific revealer except the possibility of communicating with other students. In the Preface I have given a short account of some of the material the waste of which, owing to this cause, has come within my personal observation. It seems impossible to teach such a man how to speak to any existing public; it would therefore be desirable to prepare a future public capable of understanding its future revealers of this anomalous kind; to put some children, at least, at the start of life, on the right side of the chasm which at present divides ordinary men from the man who spontaneously sees natural forms as expressions of creative thought. This can be done by accustoming the child to evoke a curve occasionally, as the outcome of his alternate sympathy with the thoughts and aspirations of animals, as described in my *Logic of Arithmetic*. And, moreover, as alternation of attitude is of far more importance than variety of apparatus and multiplicity of subjects, a child who is learning vegetable morphology by ordinary methods in the botany class will be very much helped to understand what he is doing if in the art class he is taught to investigate vegetable form by some inverse method such as that described in Chapter II.

It is not claimed that the cultivation of the scientific habit makes very specially learned people, nor that it makes people specially pious, or specially good citizens; it is simply the condition of the scientific discoverer. It may be that a man becomes more erudite, even about the subjects commonly known as 'scientific,' by learning what has been already discovered by others, than by training himself in habits favourable to discovery. It may be more useful to society to know how to do well something for which a demand already exists, than to know how to discover the Unknown. All that is here claimed is that the most likely man to discover the unknown is the one whose mind falls spontaneously into the scientific rhythm at the touch of a strange phenomenon, an unknown fact, or a new emotion.

But it should be pointed out that the scientific habit is not necessarily unfavourable to either erudition, piety, ethics, or good citizenship.

The process of original discovery, especially in pure or applied mathematics, has a peculiar effect on all the important nerve-centres; an effect which should be taken into account by all who are interested in questions connected with nerve-health, mental development, moral sta-

bility, or national sanity. The nature of this effect is explained by Gratry¹, and more fully by myself². It appears to be intelligible to specialists in medical psychology, though the mass of the medical profession seem to have, as yet, paid little attention to it.

The formula known as Boole's Equation, $x(1-x)=0$ (Boole's *Laws of Thought*), is that of the peculiar mode of brain-sequence involved in the process of scientific induction. If the brain is accustomed early to work according to the laws of induction, it acquires the power of storing up and stowing away large quantities of information, with comparatively small fatigue or injury to health.

Scientific induction is an art which, if learned early, does not lead to weariness or disappointment; it remains fresh from youth to age. This goes far to prove that it is in itself normal and wholesome. If, however, a youth has been taught scientific principles *ab extra* in childhood, and then finds his way suddenly to the bliss of personal discovery, the consequences are sometimes disastrous; like the men in Plato's Cave, never having seen anything except the shadows cast by shams, he is dazzled, blinded,

¹ Gratry, *Logique*, Vol. II.

² 'Suggestions for increasing Ethical Stability.' *Monist*, January, 1902.

and overwhelmed by the sudden contact with that reality which was his natural birthright.

Many forms of tragic waste of young life might be avoided by accustoming children to take the joy of scientific discovery quietly, as they take those other analogous delights : fresh air, exercise, and sunshine. The early cultivation of the scientific imagination on mathematical principles has other advantages besides that of preventing the future risk of danger to moral stability in case of the youth finding himself too suddenly in open daylight. It enables him both to follow easily other people's discoveries when he has time to do so ; and also to use them freely, when he does not understand, without any sense of confusion or jugglement.

The immense amount of discussion which has gone on over questions of detail about the best modes of teaching, and the best books to use, tends to blind us to the main question at issue, which is :—are our children to learn science as people used to learn classics, by permission of a privileged caste of men and books who have a monopoly and know all about it ? or, are they to learn it by right divine, because they are children of an Unseen Teacher, heirs of His kingdom and at home in His house ; and because they therefore have a right to use His

tools and His toys, His methods and His forces, subject only to such restrictions as He Himself has laid on them? Let us decide that question first; and then details about what books to use, and how to teach this or that, will settle into their proper places. In science at least, if nowhere else, the ancient dictum is certainly true: it is vain that we make our children haste to rise early and late take rest and devour many carefully compiled textbooks. If their relations with the Laws of Nature are harmoniously established from the beginning, knowledge will be given to them even while they sleep.

Children have a right also to a share in that still higher and purer delight, self-effacing communal Research. The opportunity for it should be provided in school. For if taken, under judicious supervision, by a group approximately equal in age and attainments, it is harmless and invigorating in itself, and the rhythmic beat between Altruism and Competition can be properly set up. Whereas, if the work of School and College is too prosaically and continuously competitive, the more promising pupils are often tempted to throw themselves eagerly into some group of adult thinkers, who overstrain and over-stimulate in them the desire for communal self-effacement.

Then comes the necessity for taking up the life of bread-winning ; the desire for worldly success reasserts itself. The consequences of this violent return to the worldly life, after too long a spell of Port-Royal-like Altruism, are not unfrequently disastrous to both moral stamina and intellectual power. It will be found a good rule to make all work which is properly *intellectual*, all which involves serious thought, either individual or communal ; and to reserve competition for those portions of time which are devoted to the acquiring of skill, accuracy, and speed, by practice in what the pupils already understand perfectly. The stimulus of competition, when applied at an early age to real thought-processes, is injurious both to nerve-power and to scientific insight.

CHAPTER II

THE PREPARATION OF THE UNCONSCIOUS MIND ¹.

IT is a common mistake to suppose that no preparation for science is needful or possible, except early teaching of what are called scientific subjects. Early attitude is far more important than early teaching.

Medical authorities have told us that consciousness resides in but a small portion of the total machinery by means of which we think and learn, and that it is dangerous and futile to over-feed and over-exercise that small portion, while neglecting that larger portion the action of which does not immediately cause consciousness. And, indeed, there is much reason to believe that the amount learned by children might be very much increased without the least injury to their health of body or mind, and with much less exertion than is now imposed on them and on their teachers, if the

¹ Most of this chapter is a reprint from the report given in the *Parents' Review* (July, 1899) of an address at a conference of the 'Parents' National Educational Union.'

cultivation of the unconscious and that of the conscious portions of the organism were kept properly balanced and adjusted to each other.

In the old days of classical education the training of the unconscious mind was necessarily little connected with school subjects; one can learn nothing about a dead language except by reading books or being told things. Children learned Latin in school; and the unconscious informing of the mind was done in haphazard fashion and by means of quite other subjects. Much of it was called 'idling,' or 'mischief,' or 'naughtiness.' The educational profession has not quite shaken off the influence of this old state of things; it should not be blamed if it has not yet realized what needs doing in the direction of informing the unconscious mind. For the present generation, at least, it will be wise of parents to assume that teachers, on the whole, err in the direction of attending too exclusively to the conscious mental action, and that they, the parents, must compensate what is lacking.

The first thing we must do is to resolve seriously that a good deal of time before the age of ten, and of the vacations afterwards, shall be resolutely dedicated to the training of the unconscious mind. We must not only discourage the setting of holiday lessons by masters;

we must also check in ourselves the tendency to overflow into being amateur professionals, so to speak, spoiling the future work of the professional teacher by premature and amateurish teaching.

Many parents seem to think that all the time is wasted for their children which is not spent in taking in consciously some special idea which some adult already understands. We must get rid of this notion entirely. A writer on education¹ has said that a human being comes into the world not chiefly to acquire knowledge or to develop his faculties, but to establish relations; and I would add that a child comes into science, not only to learn facts and to develop the faculty for doing things, but primarily to establish relations with the laws of nature, by which we mean—if we truly mean anything—the laws according to which the world is governed. And in order that relations may be properly established, the adults who are directing the child must, at proper times, keep silence even from good words.

I fear we are in some danger of forgetting, in the rush of modern education, that conscious mental effort rather interferes with the work of establishing relations. The time for establishing relations is the Sabbath of the I Am, the Jubilee

¹ Miss Mason.

when the land is lying fallow. Sabbath does not mean any sort of conscious exertion. But on the other hand it does not mean useless idleness. A mathematical writer on logic¹ of the nineteenth century wrote, that to listen to the voice of the Eternal Teacher we must make silence from conscious learning or even thinking; and adds, 'In these days we need repose far more than we need work. Repose is the brother of silence. We are sterile for lack of repose far more than for lack of work. The wise man acquires wisdom during the time of his repose.' A mathematician of the eighteenth century said that Sabbath and Jubilee mean, not mere cessation from work, but renewal. Sabbath, Jubilee, Holy Days, Holidays, mean, in fact, time to renew our force for future work by getting our relations with unseen forces, with nature, with man, and even with tools, more true, more perfectly harmonious, more elastic and easy than is possible while the conscious mind is acting on the relation. Begin therefore as early as you can to set up in the child's mind what one may call a Sabbatical rhythm in science; a clear distinction between the time when he is being taught by man and the time when he is free to investigate or experiment as he pleases. Give him limits of

¹ Gratry.

time and place, lay down certain necessary negative conditions for safety and health, and to avoid annoying other people; and let the child realize that during that time, in the allotted place, provided he conforms to the prescribed conditions, no one will interfere with his experimenting exactly as he pleases.

It is curious and painful to observe how many things have been proposed by true educationalists, simply for the purpose of ministering to the action of the unconscious mind, and afterwards perverted, by persons possessed with the teaching mania, to the purpose of stuffing into children's minds some idea which is in the teacher's mind. This is especially the case in regard to early kindergarten work. Each object is catalogued as intended to teach this, or to prove that, or to illustrate so-and-so; many parents seem to have no idea that it may be well to let a child have things and handle them, without any one talking, and find out what the things have to say.

We have now to consider three points connected with science:

(1) What kind of teacher is likely to err least in the matter of neglecting the unconscious cerebration?

(2) Assuming that many teachers will err in that direction, and that the parents wish to

utilize the time before school age, and the holidays later on, for the purpose of compensating this one-sided action of the school; how shall they decide, in any given case, on the kind of subject most likely to be useful in this respect?

(3) How does one set about to direct the action of unconscious cerebration, in science?

First. What kind of teachers do least harm in the way of neglecting to provide material for unconscious cerebration in science? Those are most neglectful of it who have 'got up' the elements of several sciences, simply in order to be free to advertise themselves as able to teach, and who have no other connexion with, or interest in, science than that. The best science teacher is usually a thorough-going enthusiast in the science itself, who, in the intervals of regular teaching, gets his pupils to assist him in his own investigations or pursuits. But that supposes an ideal condition of things; and ideal conditions can seldom be secured. As I said, the responsibility for unconscious preparation for science lies with the parents at present.

This brings us to the second question:—How shall the parents decide for any given family what subjects can be most usefully employed as food for the unconscious brain?

Choose for this purpose some subject to which you see the child attracted; and one the materials for which are at hand or can be procured without effort, or strain, or fuss. The means used for feeding the unconscious brain should be as far as possible dissociated in the children's minds from any notion of doing things *for their own instruction*. Whatever you set children to do for this purpose should be done either to amuse themselves, or, better, to amuse some one else; or by way of helping some one else; and by means of objects which you can introduce into the house for purposes of play, or use, or ornament. If there is any one about, a relative or intimate friend, who knows some science subject well and is not a professional teacher—an uncle electrician or photographer, a friend who owns a star telescope, an aunt with a hobby for collecting seaweeds or growing ferns, a nice friendly carpenter or blacksmith in the place where you go for holidays, let the children spend as much of their spare time with this person as they and the person wish. Lay no stress on their learning any special thing from him or her; leave the children to absorb whatever impressions they can gather. A country-bred girl who in her own childhood went nutting and blackberrying, or worked in a little garden, may be a better

holiday companion for children than the town-bred, school-trained governess, however much the latter may know about 'ovaries' and 'stipules,' and the names of classes and orders.

Thirdly, when you have decided on your subject, how shall you proceed to direct the action of the children's unconscious brain towards it?

We will begin with the preparation for Natural History. Choose as pets animals to whom you can give some sort of real natural life. Give the children something to do in connexion with the animals. If you cannot entrust the feeding or cleaning of the pets to the children, you can at least make it their business to give the horse his daily treat of fruit or sugar or bread, the dog his bath or swim. If you have birds in cages—a thing which I cannot believe is right where there are children, as I think it gives them a false start as to the *rationale* of our power over the lower creatures; but still, *if* you do—let the children accustom the birds to fly about the room and perch on their hands and heads. Let them make a garden for the cage-birds by sowing bird-seed and chickweed in pots, so that they may watch the birds picking green shoots as they grow. But it is far better to let them feed and tame wild birds.

I am not going to join in Michelet's protest against accustoming children to masses of cut flowers ; but I will say that there is no use in trusting to cut flowers and exotics raised by gardeners as preparation for understanding Nature. Let the children have such homely plants as thrive without very skilled care, and attend to them themselves. Let them also grow such things as mustard and cress for the family use. I wish to call special attention to the advisability of children doing such things as a contribution to the family's resources. It is advisable, not only on moral grounds, as tending to promote unselfishness, but because it makes it easier to secure repeated performance of the same task, with slight variations. Incessant novelty stimulates the conscious brain too much ; monotony tends to dull the whole brain ; but a duty which has to be done under varying circumstances, a uniform result which has to be properly attained under varying conditions, does much to furnish with material the unconscious brain. A child who supplies the family with small salad at stated intervals has to water it in dry weather, shelter it from very scorching sun, grow it in pots in town, and in the hardest frosts on a flannel near, but not too near, the stove ; he thus becomes accustomed to the feeling that Nature's unvarying laws of

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growth present multifold difficulties to finite man.

We come now to preparation for the physical sciences.

If there is a seltzogene in the house, let the children see it made up; not once only, with an elaborate explanation of the action of acids on alkalies—*that* appeals to consciousness; let them see it done as often as they wish, till they become saturated with the sense of the invariableness of the action; till they are sure that, though mother can make mistakes, the chemicals never do, and that when anything goes wrong it is because the human agent has been wrong.

The habit of using tools quite experimentally on a variety of material will be useful. The science of mechanics deals largely with resistances and strains. When the teacher begins talking of these things it is advisable that his class should have ready a basis of sub-conscious experience of the resistance of various woods to the hammer and saw. If you turn the children loose in company of some nice person of the artisan class, you may wish to make some compensation for any waste of his time the children may cause; let it take the shape of an informal present at the end of vacation, not of a weekly wage. The man who is hired to teach at a

fixed wage feels bound to teach ; he is sometimes tempted to treat pupils as if the business on hand were learning a lesson ; to give them the right kind of wood to make each object of, cut ready into the right-shaped block to begin upon ; to tell them in what order to do the various portions of the work, and which tool to use for each operation and how to handle each. It is desirable that children should sometimes be free to experiment under varied and accidental conditions, to use the wrong kind of wood and the tool which is not quite the best for the purpose, and hurt their fingers a little, and learn by making mistakes, with some one about who will protect them from seriously wounding themselves, will quietly prevent their making overmuch use of tools too heavy for their small hands and therefore likely to injure the flexibility of their muscles, and to whom they can apply when puzzled or discouraged.

There is some connexion between the due feeding of the unconscious brain and the process of going *wrong* ; by which I mean going at first some way which is not the ultimately right way. The nature of this connexion is as yet obscure, but it evidently exists, and we have to deal with it. Parents are given to children in order to prevent their going wrong in ways

which will compromise their future ; we must not, more than we can help, let them permanently injure their health, or acquire habits which will handicap them physically, mentally, or socially, or grow up ignorant of things which they ought to know. But the more careful we are in these respects, the more, not the less, we need to compensate the lack of wrongness in serious matters by providing abundantly safe opportunities for going wrong, and learning by experience, in matters of no consequence. For among all the habits which science requires us to form, none is more important than the habit of learning when there is no man to teach us, of profiting by our own past errors, of rising on stepping-stones of our mistaken selves to correcter judgements. Now there are few places in which a child can do so many things wrong, without injury to himself or annoyance to any one else, as in a carpenter's shop. He can begin to make something out of wood that has a flaw, or that is too soft for his purpose ; or he can try to gouge out a piece that is too hard for anything but a very sharp chisel to bite into. He can begin on too small a piece ; he can begin without taking proper measurements and put his centre-bit in the wrong place ; and, when he finds himself baffled, he can try again another way. And when he is tired of failures he can

ask the carpenter how *he* begins ; and that is a useful lesson in modesty. And he can get so delightfully dirty without any real soil or filth. But, if not in the carpenter's shop, then elsewhere, provide somewhere, somehow, opportunities for children to go wrong and make mistakes, under the protection of some one who will not interfere with them till they ask for guidance, unless serious mischief is threatening.

As preparation for hydrostatics, let the child dabble in water, with hands and feet, in warm water and cold, in salt water and fresh, as much as is safe from the health point of view. Let the baby have things to float in his bath, sticks, shells, toys of wood and china. Let him turn the water-tap on and hold his hands under it and experiment on making splashes of many shapes and kinds. I do not mean that you should tolerate such disorderly mischief as turning taps on the sly and flooding the house ; that is bad training for the child as well as inconvenient for the household. But when you are by to see that no harm is done, let the child turn the water-tap when he wishes ; not once in order that you may show him something that you can see happen, but habitually. Let him play with falling water. What is wanted is to get his finger tips, so to speak, quivering in response to the tremor of water at various

temperatures and densities, and moving in various ways. All these physical experiences pass up to the brain and produce some impression there. They do not constitute knowledge ; a man may dabble in water all his life and remain ignorant of hydrostatics as a fish ; but they do form the unconscious material which, when he comes to study hydrostatics later on, will make his knowledge living and real, not shadowy.

When a child's attention is attracted by any unfamiliar occurrence, especially by any such sound as that of a singing flame, or by a moving light on the ceiling, do not unnecessarily distract him. As long as he is interested and happy, leave him alone. Let him acquire the habit of quiet and silent observation.

As preparation for learning electricity, do not be satisfied with once showing the child that sealing-wax rubbed on flannel will attract bits of paper, but let him have a stick of wax, or better, a common vulcanite comb and a piece of flannel, and keep them, and try all the experiments he wants to try. Let him learn by experience that after a time the comb discharges and needs to be rubbed again ; that if he touches the table with the charged comb it discharges at once and he has the labour of rubbing over again. As soon as he can be

trusted to handle a glass rod without cutting himself, let him have one and an old silk handkerchief. Do not attempt to explain why the comb must be rubbed with wool and the rod with silk ; but let him find out that so it is. I have seen a charming set of toys made (from receipts published by Tyndal, for poor boys) out of paper and pith, wire and scraps of sheet tin, some sealing-wax and a few needles, with which two children, aged three and a half and five, played the whole afternoon. The habit of using them seemed to have evoked in the small mites a deftness of touch on apparatus, and a sort of personal acquaintance with what scientific people call the 'behaviour' of electric force, its manners and customs under a variety of conditions, quite different from any knowledge that would be imparted by any kind of teaching.

Of course the child has a magnet ; but he too often uses it only for attracting the regulation ducks and fish ; he should be provided with a box of iron filings and some small nails. A second magnet increases the range of free experimentation.

Most children delight in machinery which moves with a slow, steady, rocking motion. Let them waste as much time as they like in watching it. A hayband twister such as is

still in use in remote country places, the spinner of an old-fashioned rope-walk, the rocking piston of a steam ferry-boat, becomes quite a familiar friend of children who have the opportunity of cultivating its acquaintance. They might have many worse friends. But a better one still is the Sympalmograph or Harmonograph, especially the old-fashioned kind, made of two pendulums swinging different ways, and holding between them a pen, which traces exquisite curves that suggest wings, and flames, and strange unknown flower-forms. If you know where lives one of these old harmonographs, cultivate the acquaintance of the owner, and get leave to let the little ones see the big, ugly, lumbering monster creating the most fairy-like beauty by simple obedience to rhythmic law. Do not preach or try to explain ; let the motion itself lay its spell on their souls and preach a sermon too eloquent to be translated into human speech.

I said that children can have worse friends than a steam piston. One of these worse friends is the person who interrupts their fascinated contemplation by saying, ' It is waste of time to watch this old slow-coach affair ; if you want to see machines, I'll take you to see one that will turn out a gross of diagrams while this lumbering old thing makes one,

a mile of rope while this makes a yard.' If a child is left to his natural instincts he will prefer to be able to follow what is going on. The love of very rapidly whirring machinery is an acquired taste, and, for a child, an essentially unhealthy one. It is bad for nerves and eyesight ; and if a child does like it, it is in the kind of mood in which, if he got into the same later on, he would not be likely to make a scientific discovery ; he would be more likely to take to drink, or gambling, or sensationalism in politics, or to startling the public with violent attacks on sacred things, or, in short, to anything in the world which is most emphatically *not* science.

A modern child must of course acquire some sympathy with the desire for rapid achievement ; but there are better ways of introducing him to it than stunning him with the racket of rapidly whirring wheels. Choose, if possible, some machine which makes little whirr, bustle, or dust ; which causes no vicious tremor to eyes or nerves ; and in which the large amount of work got through in proportion to the amount of force exerted by the operator is due not to any piece of the machine moving quickly, but to the fact that every touch of the operator's finger sets a great variety of parts moving, each one at a moderate pace but exactly in harmony

with all the rest. The Linotype composing machine is admirable in all these respects ; and the amount of work done for one movement of the compositor's finger might fairly be called miraculous.

There is at South Kensington a selection of machines well chosen for educational purposes. Some of the models are only set working by special request made beforehand ; but several work at stated hours daily. The most attractive to children are those which they can turn on themselves by touching an electric button. When you are at the Gallery, do not be in a hurry to explain ; do not talk unless the children ask questions ; and do not imagine that the afternoon has been wasted because you have no proof that any special thing has been learned. The sensation of putting one's finger on a button and seeing a whole army of wheels, cogs, levers, and hammers respond, as if by enchantment, to one's touch, is a tremendous revelation to a child's sub-conscious mind : and, until the sensation is quite familiar, it ought to be undisturbed by any conscious teaching. The day when a child receives any great new revelation of his own relation to unseen forces should be treated as a Sabbath, a Holy-day, and no work of mental effort should be imposed on him that day.

There are three volumes, called *La Science Amusante*, written in French—easy and very beautiful French—by some one whose *nom de plume* is ‘Tom Tit.’ They supply suggestions for a great variety of experiments which can be carried on at home, and which seem to be admirably selected for the purpose of training the fingers, and through them the unconscious brain, into harmony with natural law.

Real and repeated experimentation with commonplace objects has a far more educative influence than a constant variety of so-called ‘mechanical’ toys. The common peg-top is a valuable teacher, and, like other teachers, should be allowed to speak to its pupil in its own language. When a top is asleep (i. e. when its rotation is perfect) it can be picked up from the ground and made to jump from hand to hand without its rotary motion being interfered with ; whereas the same action stops the rotation of a top which has not yet settled into its spin or which has nearly exhausted its impulsions. There are some conceptions in physical science which present no difficulty to one who, years before he hears any discussion about matter and force, has made a long series of experiments with the same object rotating under varied conditions. A useful toy for illustrating the interaction of opposing forces is the sling, which

suddenly exhibits tangential motion at the moment when the stone escapes from the counteracting force of the string. A small weight fastened to an elastic string represents the dual force under which the planets move, the force of attraction alternately yielding a little to the tangential impulse and slightly overcoming it.

The practice of playing with such toys as the sling and stone, the sucking-valve, the old-fashioned rope-maker's wheel, and the bandalore, may be made a means of accustoming children's nerves to the feeling of Nature's opposing tendencies, and may prepare the organization for receiving knowledge, later on, into the conscious brain. By training the hand to trace out Nature's action, we train the unconscious mind to act spontaneously in accordance with Natural Law ; and the unconscious mind, so trained, is the best teacher of the conscious mind.

The delight that most children take in swinging is the outcome of natural physical craving to be in line with Nature's forces. This purpose is best served by the pace at which and the height to which the child is swung being regulated according to his own instincts. If we excite him to allow himself to be swung higher than he likes, either by competition with other children or by the

injudicious comments of adults, we do something to turn what should be a preparation for future revelation into a preparation for future vice or brain-disease. The element of competition has its place, and a very important one, in education ; but its function is to stimulate to active exertion among known facts. Children may legitimately run races for a prize, or compete together as to speed and accuracy in working out sums by methods already quite familiar ; but whenever the business on hand is revelation, whether by learning to understand a mental process, observing a natural fact, or tuning the nervous system into harmony with Nature's rhythmic law, the idea and feeling of competition should be entirely absent. Too much stress cannot be laid upon this point. The false crossing of influences, the applying of stimulus inappropriate to the particular function, especially the dragging of the element of competition into the wrong portions both of the educational life and the professional life, are responsible for more of insanity and vice than any one dreams of who has not given very special attention to the subject.

Of arithmetic and algebra I shall say little here, as they are treated in another chapter. Only one point I will lay stress on. Many a life of intellectual muddle and intellectual

dishonesty begins at the point where some teacher explains the rule for Greatest Common Measure to a child who has not had the proper basis of sub-conscious knowledge laid in actual experiences. Therefore, if you value your child's future clearness in science, trust no teacher to tell him anything about G. C. M. or L. C. M. till you have ascertained that he is able ¹ to find, easily and accurately, by means of compasses, the longest length that will repeat exactly into each of two unequal given lengths, and the shortest length into which each of two given unequal lengths will fit.

We now come to the subject of geometry, the condition of which affords, it seems to me, a standing warning against directing educational care too exclusively to the conscious mind, and neglecting to provide food for unconscious mental action.

There seems to be evidence that, in ancient times, all people in good society were expected to know simple truths about geometric forms, in the same way as we all know simple facts in natural history. The elementary properties of the triangle, parallelogram, circle, ellipse, and spiral seem to have been familiar to ordinary people. They were not expected to know much about geometry, but they were expected to

¹ See *The Logic of Arithmetic*.

have and to use ordinary faculties of observation on facts within every one's reach. Euclid was, in his day, a sort of Darwin of geometry. He wrote not a geometry for beginners, but a book about the logical concatenation of geometric facts for men already geometers; just as our Darwin wrote a book about the concatenation of biologic forms for people already biologists, to the extent at least of knowing that horses prance and dogs bark and wag tails; that worms creep and birds fly; that some flowers have scent; that some fruits are sweet and others are sour.

Euclid's book was a type and model of all that a good book on logical concatenation should be. The use which was made of it till lately is the type and model of how such a book should not be used. Teachers assumed that the excellence of the book gave them the right to use it in defiance of all the laws of psychology. The result of such misuse is always the same: *loss of natural instinct*. Textbooks are written expressly on purpose to inform the *consciousness*. A good textbook should explain everything step by step, and should assume nothing which it does not actually state. Euclid does this in perfection. He wrote, as I have said, for men for whom the words triangle, circle, parallelogram were

already charged with associations ; and he gave definitions intended for the purpose, not of telling something fresh, but of clearing up and settling conceptions which were hazy from long familiarity.

Now when it became customary to give to boys of ten or twelve what Euclid wrote for grown men, that was not far wrong ; boys now can quite well assimilate what was grown-up food two thousand years ago. But if children of twelve are to learn what Euclid wrote for advanced men, children of three should be acquiring the sub-conscious physical experiences which lads in Greece picked up in the course of nature and by the accidental help of architecture and statuary. This precaution our grandfathers entirely omitted. The effect was somewhat similar to that which would be produced if it ever became the fashion to make children learn theoretic natural history from books illustrated by flat diagrams, before allowing them to see any real animal or plant. *Europe has lost geometric instinct and the habit of geometric observation.* All of us at this time are in a condition of artificially paralysed geometric faculty ; and now the aim and study of all true mathematicians is to restore the vitality of geometric instinct.

One partial remedy that has been suggested,

by Spencer among others, is to substitute for Euclid some book of similar kind but less perfect of that kind ; some book which mixes up a little real geometry with Euclid's idea of logical concatenation. This does not go to the root of the matter. The remedy is not to substitute for Euclid some inferior and less thorough textbook, but to precede and supplement the use of textbooks by some gymnastic calculated to restore normal vitality to the paralysed natural faculty. A very great deal has been done of late years, by mathematicians, in the way of suggestions towards the creation of such a preparation. The misfortune is that our mathematicians do not yet know how to explain their ideas intelligibly to non-mathematical mothers and lady nurses.

What is emphatically *not* wanted is that unscientific mothers and nurses should learn something about geometry and teach it before the school age ; what is wanted is that we should deal with those type-forms which are the subjects of geometry on the same principles as we ought to deal, and to a great extent do deal, with the other classes of form : the living forms evolved by nature, and the artificial forms created by man for his use, such as furniture, domestic appliances, and ornaments.

First comes the education of the senses.

From the time when an infant begins to stroke the cat, to smell flowers, and to handle a spoon, have geometric solids as ornaments or toys, so that the senses of sight and touch may actually develop in contact with true type-form.

Next, the training of associated ideas. When you purchase type-forms, have the correct names written on each, and take care to call each by its name, so that the children may grow up with well-formed groups of associated ideas clustered round the words which mathematical teachers will use. Be as careful as possible not to misuse mathematical terminology in daily talk ; either use it accurately or not at all. For instance, do not talk about the 'centre' of a long table, nor say 'ellipse' when you mean the oval suggested by two intersecting circles.

Then comes the training of the executive faculty. When the child can handle a pencil firmly and has outgrown the stage of mere scrawling, when he begins brush-drawing of flowers, or the drawing in pencil of boats and houses, give the hand also some training in the production of type-forms and the use of geometric tools. A violin, by the fact of being played on repeatedly, ripens and mellows into fitness for making music, because a relation is gradually established between the wood and

the musical scale ; and so it is with a child's brain ; when he is generating type-form, some relation is being established between the brain and the laws which govern the generating of curves.

Lastly—and this is probably the most important preparation for future living comprehension of mathematical ideas—there is the cultivation of the geometrical imagination. At the same age at which the child begins to realize that a tadpole grows into a frog, a boy into a man, a seedling into a flowering plant, let him have the opportunity of watching also how one geometrical type-form grows out of, or flows into, another. A common night-light placed in the bottom of a deep round jar in a dark room throws on a sheet of cardboard held over it patterns of conic sections, which pass into each other as you change the position of the cardboard. Children very early learn to love watching figures thrown in light ; and there is no age at which this amusement can hurt them, provided that the motion is slow, and that no one excites them by trying to explain things. A variety of other methods for training the geometric imagination at a later stage will be dealt with in a future chapter.

I am happy to be able to inform busy mothers that at early stages the needle and thread has

many advantages over any other implement yet devised ; a child can ornament cards by setting long straight stitches in a way which causes beautiful curves to grow under his hands without his knowing why or how, and without any pattern being set for him.

In the kindergarten attached to Bedales School the children generate designs suitable for tiles or carpets by the grouping and intersection of parabolas, curves of pursuit, &c.

In many of these designs lines are first drawn which represent the ribs of some natural leaf ; and these are then used as co-ordinates, by means of which a leaf-outline is evoked as the envelope of a system of tangents drawn in silk. Some of these designs have been noticed in the Inspector's report that he had 'found that the teaching of the calculus was leading to a most interesting evolution of design.' The beauty of some of the designs is unquestionable ; and there can be no second opinion about the value of the method, as training, from the point of view of geometry as well as from that of art. What is not quite so obvious at first sight is its bearing on the training of the unconscious mind for science. Without the slightest intellectual strain it puts the children through that normal sequence of orderly attention to classification and detail, interspersed

with nodal points of synthesis, which may be called the very breathing-rhythm of the scientific discoverer.

But to make this exercise of any use there must be no copying from diagrams ; the value of it depends on the child evoking a curve, watching it growing, under his fingers, from mere obedience to a law.

When children are introduced by any such method as the foregoing into the laboratory of the great creators of scientific thought, three classes of comments are made by adults who do not quite understand what is going on. The first takes the form of a congratulation on the cleverness of children at the present day. 'I did not know what a conic section was when I was sixteen.' The second, of alarmed remonstrance. 'Isn't it dangerous to worry little children's brains with such learned subjects ? It is far better for a child of seven to be stitching round the outline of an animal or a house on the ordinary kindergarten cards, or even working a sampler as our grandmothers did, than to be stimulating his brain with abstract mathematics.' The third comment is one of scornful incredulity. 'Now, do you believe those children understand what a parabola is ? Somebody must have shown them how to work it ; the teachers are only showing off.'

I will answer these comments in order :—

There is no more cleverness in a child drawing a curve than in the pendulum of the harmonograph doing so ; in both cases alike an implement has been set moving according to a certain law, and beauty has resulted, not from understanding but from obedience. As for any strain to the brain, stitching a curve from its tangents is not more straining to the brain and nerves than stitching round a printed outline, not nearly as much so as working the fine cross-stitch samplers of our grandmothers. The difference is that working a sampler and following a ready-drawn outline cultivate only neatness and dexterity, but the act of evoking a curve 'out of the everywhere into here,' by simple obedience to a rhythmic law, lodges an impression on the unconscious mind which will be ready to surge up in ten years' time, and perhaps make some class-teacher at College wonder why this boy or girl, though not very studious and full of all sorts of interests outside the curriculum, never had the least difficulty in grasping the idea of the differential calculus. As for the implied suggestion that there can be no other alternative except either conscious understanding or else dishonest 'show-off,' let us notice this : If the child had grown some cress in his garden, no one would have asked

whether he really understood the laws of evolution, or whether he was only 'showing off'; every one would know that he neither made the cress, nor thought he made it, nor pretended to make it; by sowing seeds as directed and watering them at due times he summoned to his aid forces of growth which he could not understand, and they responded to his call and they made the cress grow. Why had he not as good a right to summon to his aid the forces which make a curve? The cress grew even while he slept; and the curve grew while his conscious brain slept. And if the one thing is not as simple, as restful, and as unpretentious as the other, it must be because some one is making a needless muddle about what might be a perfectly normal element of child life.

CHAPTER III

HYGIENIC SEQUENCE IN DEVELOPMENT

As I shall have to speak of artificial as contrasted with natural movements, of tools (physical and intellectual) which it is natural to use as contrasted with those the use of which is artificial, I must begin by explaining that I entirely recognize the principle 'habit is second nature.' At any given stage of an individual's progress an action is natural to him if he is already so accustomed to it that the whole series of movements necessary for carrying it out follow each other spontaneously at the command of one single effort of will. The principle which I wish to illustrate is that actions which are artificial should be practised in connexion with ideas which are familiar; and new ideas should be learned by means of actions which are natural. E.g. we ought not to try to teach a little girl to cut out a doll's frock as long as she has to stop with the scissors in her hand and think how to open and shut them. We let her learn the series of movements involved in the act of cutting by

reference to some idea already familiar to her, such as that of dividing a piece of paper in two. The normal series would seem to be of this kind: The child is born with the instinct to desire food already perfected. This instinct gradually trains and co-ordinates the movements of the hand; the child learns to grasp things and put them to its mouth. When this process of grasping and guiding the hand is familiar and automatic, the child associates it with a fresh idea, that of controlling and changing the condition of things; first she only moves things about; then some day, when she is grasping a piece of paper in both hands and shaking it, the paper gives way, and becomes two pieces. If you watch the little face when this happens you see how great a revelation is taking place in the mind. The fact of man's lordship over things has burst in upon the infant brain. This revelation has taken place in connexion with a series of muscular movements already natural because co-ordinated. When the new idea of destruction has become quite commonplace and familiar, we use it to teach the little creature the manipulation of scissors, by setting her to cut (instead of tearing) paper. We allow her to cut for mere cutting's sake, till the single thought 'I will cut' suffices to place her fingers properly in the

holes, to initiate the movements of opening and shutting, and to direct and steady those movements. Till this is accomplished the use of scissors is artificial ; and, while it is so, we do not puzzle the brain with any new idea in connexion with scissors ; they are used only in doing what the unaided fingers did before—severing the paper. Not till the process of cutting has become a natural one do we introduce the new idea that destruction is not to be merely destructive, that it can and should be directed towards some definite aim, subordinated to some constructive purpose ; at no step in this sequence is the mind distracted by trying simultaneously to receive a new idea and to correlate for a still artificial process.

I believe that hardly any mistake in education is more disturbing to normal brain-action, more likely to induce nerve-storms in delicate children, or more dangerous to future brain-power in all children, than the attempt to convey a new idea by means of a process still artificial (i.e. inadequately co-ordinated) or to teach a new process by means of an idea still unfamiliar. Another aspect of the same principle is expressed in the homely adage ‘Fingers before forks.’ The word ‘before’ of course is here a pure adverb of time ; it has no reference to ultimate preference. None of us

wish our children to grow up savages and grasp mutton chops in their fingers ; but none of us teach a child the use of a fork until after he has acquired the art of holding a bit of bread firmly in his hand and carrying it steadily to his mouth. We do not attempt to teach him the use of artificial tools for extending the scope of any action, or for refining the method of it, until the movements necessary for performing that action at its more elementary stages have been not only learned, but also perfectly co-ordinated by long practice. Think what an oculist would say if we taught a baby to use optical instruments before the focussing apparatus of its eyes was under proper automatic control ! He would warn us that a child so treated would not be likely ever to get the best use of either simple or artificial vision.

So far as physical tools are concerned, it is quite admitted as a principle of Educational Psychology that there should be a long interval between the first suggestion of a new idea and the use of special tools for carrying out that idea into its late ramifications. It is also admitted that the use of new tools must be learned by reference to ideas already familiar ; and that application of the tool to carry out complicated ideas should be suggested only after the actual manipulation of the tool has

become quite familiar and automatic. Nor is any one, as far as I know, careless in carrying out the principle where the physical tools are concerned. I have yet to meet the mother or nurse who would set a child to cut out a shape the first time it was trusted with a pair of scissors, or who would allow it to get its first notions of tune while learning to hold its hands properly on the piano. But there are such things as intellectual tools and processes. Unfortunately Educational Psychology has not yet got a firm grasp of the distinction between intellectual tools and processes and the ideas worked out by means of them. It is hardly yet sufficiently understood that there is as much difference between a scientific ideal and the process by which we realize it as between the idea of a frock and the process of making it; that there is as much difference between a mathematical principle and the formula in which it is registered as between a loving message and the paper on which it is written.

To make clear what it is that is so chaotic in this matter, let us consider the question so often asked, 'When should a girl begin learning Elocution in the dramatic sense?' The answer should be, When, besides understanding the construction of sentences, she speaks and reads simple books well, when she has

complete mastery of all the elementary sounds in the language, can pronounce each of them singly and purely, and can make them staccato or prolonged at will ; and can also make readily any desired sequence of two or three sounds. The processes to which I refer (correct articulation, &c.) are the tools which dramatic elocution must use from its beginning ; therefore they should be perfected, before it begins, by carrying out the reading of simple tales and recitation of easy poems. When the pupil has, by means of them, arrived at some progress in expression, it will be time to teach other processes for carrying dramatic effect to a higher pitch. But, as things are, parents send a girl to a Shakespeare class because ‘she reads so badly, and it is such a nice accomplishment to be able to read aloud well.’ The poor elocution teacher is expected to jam together exercises which belong to quite different periods of development. How can a girl take in the sense of a drama, or learn to express its emotions, when the emotion itself is interrupted at each moment by instructions about how to move the jaws and tongue to produce a particular sound ? The relation between the form taken by the mouth and the sound produced is in itself an interesting study ; but who can understand it well enough to be interested if

she is being nagged at about how to express the emotions of a lover in a play? The elocutionist has to instil habits which ought to have been acquired before the child was six, while teaching her to recite effectively poems which she should not read till sixteen. Who can say at what age this ought to be done?

The mathematical formulae which some people speak of contemptuously as 'dry' are in reality as beautiful as microscopes, or any other well-adjusted and well-finished machinery intended to extend the scope of man's powers; and as for mathematical ideas themselves, they are as grand as any expressed in poetry. Comparatively few people get any real enjoyment out of either; and the reason obviously is that their faculties have been stamped into confusion by a method which I can only compare to that of making a child use a telescope before it can see properly with its eyes; using a complicated machine for extending and refining certain work before he has learned to do, without it, the simpler kinds of that work.

To make clearer what it is that is wrong, let us think of an orchard, at harvest time, which contains, besides the trees and fruits, the natural human limbs with which man picks the fruits he can reach; the stools, steps, and ladders of various lengths, which are an exten-

sion of his own legs and by means of which he rises to the level of the fruits which he cannot naturally reach; the sickles or fruit-scissors which improve on the action of the hand and enable us to bring down the choicest fruit without risk of spoiling its delicate bloom; the room fitted with shelves on which the fruit is stored for future use; and the baskets in which it is temporarily packed for safe conveyance to the store-house. It is no exaggeration to say that all these various items have analogues in science, especially in mathematics. A mathematical textbook contains truths valuable in themselves because throwing light on the nature of human thought; other truths valuable for use in commerce or physical science; natural processes of reasoning by which the student can gain direct perception of some of these truths; artificial devices for arriving at others; refinements of various sorts; and formulae and tables in which truths are classified, stored for easy access, and preserved in the memory. It is pitiable to see how far are many students, even advanced ones, from any clear realization which of these various things is which. Many have no conception of the difference between direct and inverse in mathematics. They make no clear distinction between the truth itself, the ladder of devices by which they reached it,

and the formulae in which they stored it. And can we wonder at this? No human being, I suppose, ever attempted to teach a child to climb a ladder, to use a fruit-sickle, or to store fruits on shelves, in the same summer in which it first was able to stand on its legs and grasp a low-growing apple, in which it first experienced the delight of eating fruit. It would probably be an under-statement of the case to affirm that a mode of treatment analogous to this has been inflicted on ninety-nine per cent. of the young people who have learned Algebra and Trigonometry even within the last ten years. The methods of doing this stupid thing have no doubt improved enormously since the early days of De Morgan; all praise is due to the patient workers who have done so much to improve them; but the stupid thing is still done; and the parents and the public still insist on its being done. The questions so often put by parents, ‘At what age do you think my child had better begin Algebra?’ (or Trigonometry) and, ‘Can you recommend me to a good teacher?’ really mean something analogous to this:—‘I intend to keep my child ignorant of all experiences concerning fruit, and all processes connected with it, till he is old enough to begin receiving straight away, in one continuous series of lessons, information, conveyed by

verbal explanations, about how to stand on one's legs, how to climb ladders, how to use sickles, how fruits taste, their hygienic and economic value, their botanical classification, and the best means of preserving them. At what age do you consider this series of lessons should begin, and whom do you recommend me to employ to give it?' The only answer one could make to such a question would be that there is no age at which any such course should begin, and no person who ought to be asked to give it. The question asked by a parent should be, 'At what age would you recommend me to let my child begin learning such portions of Algebra (or Trigonometry) as can only be learned by the aid of complicated devices invented, centuries after the science itself was an actual working possession of our race, for the sake of projecting its action into fields which would be inaccessible to it if only natural and simple tools were used?' The answer should be, 'When the process of learning by the more direct means has become so familiar as to be performed sub-consciously.' The time to begin learning numeration on paper by means of digit-places is when the child not only knows but is familiar with the conception that a red counter may stand for ten white ones, and when he can count red ones 'ten, two-tens,

three-tens,' as easily as he counts white ones, 'one, two, three.' The time to begin 'proportion sums' is when he saves himself trouble in measuring or counting by the method of proportional measurements. The time to begin doing long sums in Greatest Common Measure and Least Common Multiple is when he is not aware of any difficulty in finding the lengths of the longest pattern that will repeat exactly in each of two unequal cornices, and the shortest length in which each of two patterns of different lengths will repeat. The time to begin Algebra on paper is when he amuses his baby sister by setting her little problems in which she has to guess what unknown number a certain coloured counter stands for; and tells her that 'long ago, when I was little, mother played that game with me.' The time to begin formal Geometry is when making circles, ellipses, and equilateral and isosceles triangles, by means of compasses, pins, and a thread, is as easy as feeding himself with a spoon. The time to begin theoretical Solid Geometry is when he has exhausted the interest of trying experiments in fitting together regular solid figures of different sorts. The time to begin the use of Trigonometrical formulae and tables is when he can no longer realize—though he perhaps dimly remembers—that there was a period of his life

when he was not quite sure how to find the height of a church tower by means of a walking-stick and a yard measure. While we are on this part of the subject I may mention that there exist books for grown-up people on the so-called 'Fourth Dimension,' in which solid shapes, such as crystals, are treated as the shadows or sections of things not visible to or conceivable by man; and that the time for getting anything except confusion out of such books is when one has a clear familiar knowledge of the true relation of the earthly shapes themselves to the shapes of their sections when cut, and of their shadows or projections on flat paper. For the majority of children the really right time for beginning any department of mathematics never comes, because the really right time is the time when the right condition of development has been attained; and for nine children out of ten it never exists at all.

Between the process of introducing a concept or idea into a child's mind and its bringing forth as an intelligible entity which the conscious intellect is to handle and reason about there should intervene a period of gestation, during which the idea is left dormant and passive in the child's mind; any attempt at premature interference is likely to result in abortion. We are not here reasoning by ana-

logy; we have independent experience of the need of intellectual gestation. I am using physical gestation merely as the readiest way to convey a clear notion what is the doctrine of medical psychology on this subject.

Parents imagine that if they have secured for their child the best textbooks and the best teacher they can they have done all that is necessary for his education in the particular branch of mathematics which he happens to be pursuing; too often they have omitted to secure something much more important than the quality of the textbook, perhaps even than the skill of the teacher; that is, a sufficient time-interval between the child's first introduction to the subject and his first introduction to the textbook and its machinery. The worst textbook in the world, where that condition is secured, will do more to make a genuine mathematician than the best without that condition. The textbooks have to squeeze together into any one chapter instructions which ought to be separated by an interval of months or perhaps years. And, moreover, the poor textbook has not only to teach what should be taught to the well-prepared during several successive years, but also to mix up this teaching with attempts to supplement the deficiencies of those crippled and cramped by lack of natural exercise.

(Therefore, when a textbook is put into the hands of a well-prepared pupil, one has to keep a little watch on his private reading of it ; a few words from the teacher are sometimes necessary to prevent his worrying his brain by efforts to find a meaning in long paragraphs which are, for him, useless rigmarole, being elaborate attempts to convey to the ill-prepared some slight and necessarily imperfect notions of things as to which the well-prepared cannot remember the time when they were not quite clear to him. This caution is important from a hygienic point of view ; the fatigue of wrestling with a long explanation, which at last turns out to be about something that needed no explaining, is often greater and always more unhealthy than that of learning what is really difficult.) As things are at present, it is almost impossible for the author of a textbook in science to avoid linking together things which in actual teaching should be separated by long intervals of time ; it is for the parents and teachers to guard against misuse of the books. My object is not to encourage criticism of the existing textbooks, but to prepare a future generation of parents who will demand better ones, and whose children will be suitably prepared for using such. We may be sure that, as soon as we have created a demand for the right

books, authors will spring up capable of meeting that demand. Meantime we can do a good deal to minimize the bad effects of the present arrangements by using the existing textbooks in a less slavish way than is commonly done. We can sort out for each child the elements of procedure which have become natural to him from those which, in his particular case, are still artificial; we can instil new ideas by means of the former, and interpose between the new idea and the unfamiliar process that time-interval which is necessary for maturing and mellowing the idea itself.

I have seen in print disparaging comments on play-methods of introducing children to arithmetical ideas. Such criticisms seem to be founded on an erroneous assumption that the play-methods are intended to teach something about arithmetical procedure, for which purpose they are, as the critics perceive, ludicrously inadequate. But these methods have no relation to the teaching of anything: their use is to give to the infant brain a start on a line of development such that, in after years, when the intellectual teaching begins, it will be capable of receiving that teaching. I should be reluctant to insist on any opinion contrary to that of more experienced teachers than myself, *in any matter about which they have the experience.*

But the majority of professional teachers, at present, on their own confession, have no idea on what depends the receptivity of the child when the teaching age for the subject has begun. Parents also are still so unawake to this that they confuse preparation for a subject with premature teaching of the subject ; and do great harm by such premature teaching. Many teachers strongly deprecate amateurish attempts to instruct children before the proper time ; and in this I entirely agree with them. A concrete instance may help to make clear in what preparation for a subject consists. In my young days cards of different shapes were sold in pairs, in fancy shops, for making needle-books and pin-cushions. The cards were intended to be painted on ; and there was a row of holes round the edge by which twin cards were to be sewn together. As I could not paint, it got itself somehow suggested to me that I might decorate the cards by lacing silk threads across the blank spaces by means of the holes. When I was tired of so lacing that the threads crossed in the centre and covered the whole card, it occurred to me to vary the amusement by passing the thread from each hole to one not exactly opposite to it, thus leaving a space in the middle. I can feel now the delight with which I discovered that the little blank space

so left in the middle of the card was bounded by a symmetrical curve made up of a tiny bit of each of my straight silk lines ; that its shape depended upon, without being the same as, the outline of the card, and that I could modify it by altering the distance of the down-stitch from the up-stitch immediately preceding. As the practical art of sewing perforated card was already quite familiar to me, my brain was free to receive as a seed the discovery I had made, and to let it grow naturally ; all the more because no one spoke to me then of tangents, or tried to teach me any algebraic geometry, till some years had elapsed. Therefore, when I did begin to learn artificially about tangents, the teacher was not obliged to put cuttings into raw soil ; he found ready a good strong wild stock of loving interest in the relation between a curve and the straight lines which generate it, on to which he was able to graft the new knowledge. The teacher came, not as an outsider thrusting on me the knowledge of something unfamiliar and strange ; but as a brother-seer more advanced than myself, who could show me how to make further progress on a path which I had already entered with delight. On such accidents as this of my card-sewing depends, I think, much of those special receptivities for certain subjects, quite distinct from great power,

which puzzle psychologists. When we understand better how they originate, they will no longer depend on accident, and we shall more often be able to produce them at will. But let me repeat once more : the receptivity cannot be generated by early teaching of a subject mixed up with the use of its appropriate technical machinery ; but only by suggesting the new ideas by means of objects already familiar to the child's eye and touch, or of processes become habitual and automatic by use in connexion with a familiar idea. Unless this and a few other principles of normal sequence, admitted by all medical psychologists, are incorporated into the basis of our educational system, it is to be feared that more improved ' methods ' will increase the danger of abnormal sequences, and will turn out to be improved methods of injuring nerve-stamina, intellectual grasp, and moral stability.

CHAPTER IV

THE CULTIVATION OF THE MATHEMATICAL IMAGINATION

THEORISTS in education sometimes imagine that a good teacher should not allow the work of his class to become mechanical at all. A year or two of practical work in a school (especially with examinations looming ahead) cures one of all such delusions. Education involves not only teaching, but also training. Training implies that work shall become mechanical; Teaching involves preventing mechanicalness from reaching a degree fatal to progress. We must therefore allow much of the actual work to be done in a mechanical manner, without direct consciousness of its meaning; an intelligent teacher will occasionally rouse his pupils to full consciousness of what they are doing; and if he can do so without producing confusion he may be complimented and his class congratulated.

We teach laws of curves by reference to certain straight lines—tangents, co-ordinates, radii, &c. These lines bear the same kind of

relation to the curves which the framework of sticks fastened into a pot bears to the climbing plant which is the true object of the gardener's care. The plant itself is living and growing; the justification for the existence of the framework consists in the fact that it would be impossible to get the true enjoyment of the plant without its aid. The co-ordinates form no part of what we want to teach about; but we cannot learn without their help. They enable us to see how the curve came into being, and whither it is tending.

Suppose then that a class, while becoming skilful in working problems, seems to have forgotten that the axes are no part of the curve itself. The teacher may wake it up by saying, 'You don't imagine, surely, that axes form any necessary portion of a curve? An ellipse, for instance, is the path of a body moving round a focus of attraction. Suppose a planet were endowed with the power of leaving a track in the sky, the track would be an ellipse only, unencumbered with axes. What are the axes, then? Indications of invisible forces? Not so. No line of attraction at any point of the orbit corresponds to the minor axis. The axes are *Human Devices* to enable us to measure and express the various elements of the orbit.'

'Well, but,' exclaims perhaps some clever

pupil, 'if the straight lines are unreal, if they mean nothing, why were they invented, and why were we made to study them?' Such reasonable criticism is a great help to the teacher. He proceeds to picture the state of Astronomy in the days when nothing was known of the movements of the planets. He describes the first bewildered effort of the human mind to represent to itself the path of these wanderers. He shows how some man may have at last conceived the brilliant idea of projecting imaginary straight lines across the sky from one fixed constellation to another, thus forming a sort of background of measuring-rods; how the constellations, with these imaginary connecting lines, might be copied on a tablet, and the path of the planet registered thereon from day to day; and how Science might grow up by man inventing modes of measuring and registering curves which the living forces of Nature were describing in space.

This should come at the end of the lesson and be followed by a few minutes of silent meditation before the class breaks up and the pupils give their attention to some other subject. In the next geometry lesson co-ordinates should be treated in a purely mechanical spirit; no fresh stimulus being

applied to the imagination. But the class will go on with its work with renewed interest and quickened intelligence.

Again, the tangent to an ellipse is an imaginary straight line, representing the path which would be followed by the body tracing the ellipse if its connexion with the attracting focus were suddenly to cease. In its essence, the tangent is a sublime effort of the scientific imagination ; it pictures the result of a sudden cessation of the action of gravity. In practice, the tangent is a convenient line for indicating the curvature at any given point. The educational sentimentalists who object to mechanicalness ought, if consistent, never to use a tangent in working a problem without stopping to realize the grandeur of the idea involved. As a matter of fact, such incessant strain on the imagination and on the perception of the sublime is unhealthy and deadening. We ought to use tangents, mechanically, as mere measuring-rods. But a good teacher will take care that no pupil goes through a year's work at Analytical Geometry without having been, once or twice, aroused to perceive the wonderful poetic conceptions represented by the lines he is using.

The above is a description of what, I suppose, everybody will recognize as a reasonable manner of dealing with a class of children, say, between

the ages of 14 and 16. The present chapter purports to show how this element of rationality can be infused into the elementary stages.

The teacher who would educate the mathematical imagination of pupils must begin by cultivating his own. It might assist him in doing this if he occasionally put before his mind a picture of a hypothetical class in Botany. Let him imagine a town whose inhabitants never have access to plants or flowers, except in botanical class-rooms, wherein are arranged microscopes ; lenses variously mounted ; forceps of various shapes ; dissecting needles and blades of many kinds fitted with convenient handles ; water-dishes for soaking specimens ; a few cut leaves and flowers, and a few plants in pots. It is no exaggeration to say that a state of things analogous to this has largely prevailed in regard to mathematics. It is needless to say that it is a bad state of things, and that reformers wish to establish an easy familiarity with geometric forms and conceptions previous to, and independent of, every sort of formal teaching. But, pending the possibility of this radical reform, some ways of dealing with the prevailing state of things are less bad than others. If a botany teacher in our supposed plantless, treeless town were to begin his course of instruction by saying to the class,

‘These objects on the table are plants ; and the movements which I shall cause to take place before your eyes will be processes of plant growth,’ he would create hopeless confusion. He would do the best possible under the circumstances by saying something of this kind : ‘A few of these objects are plants ; most of them are ingenious human devices for assisting man in satisfying the peculiarly human desire to learn about things which escape his direct cognizance. If we were angels or any other creatures possessed of super-human powers, we might be able to take cognizance of growth-processes in a direct manner ; we might see sap flowing in a branch and leaving deposits of fresh cells. But we human creatures cannot see growth-processes going on ; we can only infer or induce a knowledge of them by comparison of various facts. Even the facts by means of which we infer or induce a knowledge of the laws of vegetation are, for the most part, hidden from our immediate view ; we can have access to many of them only by tearing plants to pieces. I am, therefore, about to violate the sanctuary of this plant’s life, to tear its structure by processes contrary to its nature, though suited to ours ; and thus enable you to see facts from which you may infer a conception of the nature of its growth. I could do this roughly

with my fingers and nails, and you could then partially see with your eyes ; but the apparatus provided to man by Nature is adequate only for the satisfaction of his elementary animal needs ; if we would gratify our higher curiosity we must invent extra apparatus for the purpose. The forceps which I shall show you in use are fingers more sharply pointed than our own ; these knives and needles are specially adapted nails or claws ; with these lenses and microscopes you can see objects too small for your unaided eyes to perceive.' I do not mean to say that any speech analogous to either of the above ever was made, or ought to be made, in words, by a mathematical teacher to his class. Confusion is created or avoided less by what we say or do than by what we take for granted. The latter of the two supposed speeches represents the conception which ought to underlie mathematical teaching : the former fairly pictures the assumption which too often does underlie it. The true remedy for the present condition would, of course, be such a general familiarity with superficial geometric truths as we actually have with superficial botanical truths ; a general knowledge such that no grossly false assumptions could prevail in a class. But failing that desideratum, or until we can attain it, the next best thing would

be a clear conception in the teacher's mind of what the thing is which he is undertaking to do.

It should be understood from the first that no such thing exists as *a* right method of performing any operation in elementary mathematics ; because all rightness, and I may add all mathematicalness, depends essentially on getting each operation performed by *two methods* ; the first, a roundabout one, which represents and registers the conscious action of the mind during the process of discovery ; the second, a short method which condenses the roundabout one, assists in stowing its results away in the memory, and facilitates the using of them sub-consciously. As an example of misunderstanding on this subject take the following :—English children used to be taught, straight away, how to do ‘ Rule of Three,’ which few of them understood. It was then discovered that comprehension of Proportion is more easily arrived at if the first exercises in Proportion are done by the so-called unitary method, which is a registration of a natural roundabout process of analysis. Now, children are, in some schools, encouraged to work sums in Proportion by the unitary method always, and are not taught any condensation. There is nothing that deserves to be called mathematics in either the

old way of teaching or this newer one. The *mathematics* of Proportion begins at the point where the pupil, having thoroughly mastered the unitary method, with full conscious comprehension, learns to condense it into some notation which he can afterwards use sub-consciously.

The cultivation of the mathematical imagination depends chiefly on the child being put into the right attitude towards mathematical conceptions in his earliest years ; and, after that, on the right use being made of certain nodes or critical points, which occur here and there in each branch of mathematics and which should be dealt with in a quite different manner from the rest of the course. These form the revelation-crises of the pupil's mathematical history ; when he draws near one of these the human teacher should carefully withdraw his influence, and simply watch to see that no seriously false impression is being formed. His object should be to efface himself, his books, and his systems ; to draw aside a curtain from between the child and the process of discovery, and to leave the young soul alone with pure Truth.

The following directions are not meant to be absolute, but rather to convey a picture of a sort of type-scheme for the use of critical points ; any intelligent teacher will easily modify it according to circumstances.

As soon as the child can count up to ten teach him to call the next numbers 'ten-one,' 'ten-two,' &c. The more usual 'eleven' 'twelve,' &c., can be easily learned four or five years later ; the habit of illogicality induced in infancy by the dislocation between the 'eleven, twelve' nomenclature and the 'twenty-one, twenty-two' nomenclature is far more difficult to overcome.

Accustom the child, when he has counted out ten counters or buttons, to put them into a little box ; the next ten into another similar box, and so on. Let him do this simply as a matter of order and routine. At first, leave the boxes open during the whole lesson ; but exercise him as soon as possible in the habit of walking by faith (in his own past work), by covering each box as soon as it has received its cargo of ten. Later on, a red counter is made to do duty as substitute for a box containing ten white ones ; then a green or black one for ten red ones ; and so on.

It is well to accustom the child to realize early that any convenient number might have been chosen as the breaking-point ; that ten was chosen originally because savages had no counters and therefore counted on their fingers ; and now he must count in tens unless the contrary is mentioned, in order to understand

and be understood, because all the world does so. Tell him that English children are not allowed to count on fingers, for the same reason that they are made to use forks and spoons at table; i. e. because it is best to learn to use civilized conveniences. This lays the foundation of a true sense of historic continuity in Mathematics; and cultivates comprehension of its anthropomorphic basis.

It is advisable occasionally to show a skeleton leaf, the cuticle under the microscope, and the cellular substance; and then appeal to the pupil to conceive these various portions, combined and acting on each other, in the growing leaf. The child should be led to see that it is not the nature of the leaf to grow its cuticle, its ribs, and its cellular substance separately, and then combine them; but it is the nature of the human mind to work by taking things apart and then conceiving of them as united. He should feel that in Arithmetic we do the same kind of thing with regard to numbers too great for us to deal with directly; that Arithmetic is the application to number of a scientific system for making difficulties manageable, by splitting them up in ways suited to the nature of the human mind. Of course one does not say things of this kind directly to a child; it is not desirable to make his mental processes intro-

spective. But the teacher should think of Arithmetic as founded on scientific anthropomorphism ; and, when the youth is old enough to look back on the course of his mathematical education, he should see himself as having been led throughout to apply general psychological laws to the solution of questions concerning number, size, and direction.

The child should be familiar, as early as possible, with the practical exchange of coins ; getting farthings in exchange for a penny (same weight in a different shape) and pence in exchange for a shilling (a more conventional equivalence). (The education of children would be more *simple* if the coinage were decimal ; I am not sure that it would be necessarily sounder.)

When he begins to do sums on paper, let him still, for a considerable period, do each addition, subtraction, &c., first in counters ; and then, while these are still on the table, work out the same sum on paper.

As soon as he can do little additions on paper, begin to cultivate his sense of the right relation toward formulae ; give a blank form of multiplication table ; give, as addition sums, two and two ; two and two and two ; and so on ; (alternating these examples with additions of unlike figures) ; and, whenever he has added

together a group of like numbers, make him register the result in his blank form. When he begins multiplication, let him learn the tables by heart in the old-fashioned way ; but from his own handwriting. Let him re-make or revise his table as often as is found advisable, but never allow any counting during the process of doing a multiplication sum. Make him use the table mechanically and without question ; never allow the process of making or verifying formulae to get entangled with the use of them ; 'verify or construct as often as you like ; but never while using,' should be the rule.

Some set of relations should be chosen to be expressed in two different ways, numerically and graphically ; and the child should be accustomed to refer, when he needs the recorded fact, sometimes to the one table and sometimes to the other.

Keep up the same habit when he begins Algebra. As soon as he can make out the results of $(a+b)^2$, $(a-b)^2$, $(a+b)(a-b)$, &c., let him do it algebraically and geometrically ; then enter each result, as he arrives at it, in a formula book. As soon as a formula is made, give him easy practice in using it, always reading off from his own handwriting ; trusting to his own past work. Let him practise re-making the formulae till the whole process is

familiar ; but do not let the making take place at the same lesson as the using.

If this system is steadily pursued throughout, it will be safe to give the child logarithm tables as soon as he is found capable of manipulating them. He will realize, in a general way, that adding the indexes multiplies the numbers : he will not imagine he knows exactly how the decimals in the logarithms were worked out ; but he will assume, and quite rightly so, that it was done in a way somewhat similar to that in which he constructed his own book of tables and formulæ. He will use the logarithm tables with the same natural gratitude to the maker of them as he has towards the maker of his watch (to use Prof. Perry's simile). This educates or educes the maximum at once of the grateful acceptance of the work of the past which the Positivist School were so anxious to cultivate, and that sense of being free of the Guild of Discoverers of which some among them seemed to have a dread.

It seems to me a great pity that teachers, who consider themselves advanced, should drop the practice of making the child repeat his tables and formulæ periodically, in parrot fashion, as a mere exercise of memory. Psychologists such as Gratry have pointed out the

importance, for both individual and racial mind-stamina, of some form of words, the same from year to year and from generation to generation, which can serve as a crystallizing thread (or, as Hinton called it, an 'unconscious constant'), round which the results of intellectual effort may successively group themselves. The two essential conditions for the full efficacy of a crystallizing thread are, that no disturbing element of doubt should cling round it, and that it should be associated with some form of solemn and yet joyful emotion. If the formula itself contains any truth worth remembering on its own account, so much the better; though this condition is of less importance than the two others just mentioned. Now a well chosen series of mathematical formulae answers all these requirements perfectly. Their truth is unquestionable. The child's stock of them grows with his growth; yet those earliest learned remain valuable and important to the end; the same for all, from youth to age and from generation to generation. And if they have been acquired in a reasonable manner, each is associated with the delight of personal discovery; and is, for the child, a perpetual pledge and witness of his own power to enter into personal communion with As-yet-unknown Truth.

It is useful to connect money payments with

the visual image of a shop-counter ; to set up the habit of seeing money paid for goods, as coin pushed one way across the counter ; and change, or money returned when a purchase is revoked, as coins pushed the other way ¹. This makes a link of connexion between the algebraic and the geometric interpretation of the plus and minus signs.

It is important to teach pupils to distinguish early between several different kinds of wrongness :—

$5 \times 4 = 19$ is a wrong statement, because untrue in itself.

$\frac{21}{7} = 3$ is wrong when given in answer to the question, What is $\frac{217}{7}$?

It is wrong to begin a subtraction sum by the coins or number-places of highest value, because it necessitates cumbersome and unnecessary corrections.

It is wrong to write what you mean for units to the left of what you mean for tens, because, if you do so, other people will not understand what you have written, and you yourself will become confused in passing from your own figures to those written by others, and will lose the advantage of forming a steady habit for reading and writing figures.

The geometric education may begin as soon

¹ See *Logic of Arithmetic*.

as the child's hands can grasp objects. Let him have, among his toys, the five regular solids and a cut cone. Let eyes and hands be early accustomed to recognize the difference between natural form (that which evolves without human aid), artificial form (that which is made by man in response to a need from without, as a table, a tool, or a toy), artistic form (that which is created by man in response to the sense of beauty within him), and type-form (that which is created by man in obedience to his mathematical sense).

Nothing is more important for the cultivation of the properly mathematical sense than the habit of evoking curves by drawing successive tangents. This can be done, before the hand is strong enough to manage a ruler, by stretching threads on cardboard. The usual kindergarten pricker, pad, and rug-needle are the only tools required for this purpose. The early use of this exercise in fertilizing the unconscious mind has already been described. It has another advantage besides that of easy manipulation which makes possible its use at an early age, one which will presently be indicated.

Great use might be made, in teaching, of the study of shadows. Those who are only beginning the shadow-study can work most conveniently with a single light overhead. Later on,

combined and cross lights can be used ; and in some cases it will be useful to have a movable light. Place on the table a sheet of white paper. Hold between the paper and the light a ring. Call attention to the fact that the same ring casts a circular or an oval shadow, or a straight line, according to the position in which it is held. Also that the same series of shadows is produced by an elliptical ring as by a circular one. Either can be made to cast a shadow resembling in shape the other. A straight line, however (a knitting-needle, for instance), cannot be made to cast a curved shadow on a plane ; its shadow is always a straight line, which shortens as the needle is tilted up, till at last it resembles a mere dot. Or if a circular disk of cardboard be held horizontal under the light, it can be made to cast a series of shadows resembling in turn each of the conic sections (circle, ellipse, hyperbola, and parabola), by altering the position of the paper on which the shadow is cast. The same series of forms may be produced by placing a lighted night-light in the bottom of a tall jar, and throwing the shadow of the rim of the jar on surfaces held in different positions. Experiments should be made with the various kinds of shadow that can be cast by a coiled wire, such as is used in electric machinery. It is

also well that the pupil should see such a coiled wire taken as a whole and coiled again into a larger spiral, as preparation for the conception of 'space diverse, systems manifold.'

A child should early be accustomed to drawing circles by means of a pencil constrained by a loop of thread hitched to a fixed pin. When he can get firm and accurate circles in this manner, two pins at a little distance from each other should be placed within the loop; then three pins. He should vary the distances of the pins from each other, and the position of the third with regard to the other two. As soon as this exercise is quite familiar he may begin drawing with trammels of different kinds¹.

It is useful to watch, in a dark room, the

¹ Trammels may conveniently be made of cardboard or thin wood, such as kindergarten plaiting sticks, in which holes may easily be made for a pencil to pass through. If a single bar be fixed to the drawing-board at one end, points along it describe circles of various sizes. A simple linkage system, producing beautiful curves, may be made as follows:—Take three bars of equal length, AB , CD , EF . Make a series of holes along CD . Fasten B to C and D to E by an inverted drawing-pin or in any other way which permits rotation. Fix A and F to the drawing-board. A pencil passed through any one of the holes and moved through all its possible positions traces out a curve. The middle point of CD gives double symmetry; any other point on CD , single symmetry.

path of a bright point. The effect can be fairly well got by igniting the end of a stick and then blowing the flame out, leaving the tip glowing, and whirling the stick about ; but the thin flexible Japanese joss-sticks answer better. The essential thing is that this shall not be an exhibition given once by an adult with an attempt to explain any theory about the resulting phenomena, but an informal series of vague experiments in which as many of the family as like to do so take part for sheer amusement and artistic enjoyment. Any explanation that could possibly be given by an adult would distract the child's attention from the real purpose of the game, which is to fix the imagination on the fluidity and mutual interchangeableness of natural curves. For the same purpose, he should also be allowed to watch natural reflection-curves, such as those made by the sunlight or gaslight falling on moving liquids, the light-curves at the bottom of a clear stream caused by the ripples on the top, &c.

If the child habitually walks out in company with a perambulator or mail-cart, it might be well to paint a conspicuous spot on the rim of one of the wheels. Nothing need be said about this spot ; it should be left simply to fascinate the child's imagination. In connexion with this question of a conspicuous spot on a moving

object, the following is an interesting example. Paint a spot on a cane. Stand the cane perpendicularly in a corner of the room. Draw the lower end of the cane away from the wall, keeping the upper end close in the corner, till the cane is lying horizontal on the floor. Let the child watch the path of the spot. As soon as he can manipulate drawing-tools easily he might attempt to draw on paper the curve made by the path of the spot. Next he should draw the cane, with its spot, in various positions, first without, and then with, the aid of a ruler finely divided for accurate measurement. A little practice of this kind would obviate the danger of future confusion as to the connexion between a continuous curve and the points taken to determine it for graphical representation.

As soon as the hands can hold steadily compasses and set-square, the child should be encouraged both in copying diagrams from such books as *Elements of Practical Geometry, Plane and Solid*, by Thomas Bradley (published for the Committee of Council on Education, by Chapman & Hall), and in inventing others for himself. It is desirable that, before any systematic teaching of mathematics begins, the compass, set-square, and ruler marked in fractions of an inch should be as familiar implements as the fork and spoon.

Valuable exercise is that of tracing the pentagram. Number the angles of a regular pentagon in order, and draw straight lines without a ruler from 1 to 3, from 3 to 5, from 5 to 2, from 2 to 4, from 4 to 1. Repeat several times in succession. Choose the size of the pentagon to suit the size of the hand. After some practice the pentagram should be drawn without the aid of a pentagon. The exercise may be varied by tracing the heptagram, i.e. passing from one point to another of a regular heptagon in this order—1, 4, 7, 3, 6, 2, 5, 1. The tracing of these figures gives a curious feeling of arriving at completeness by a series of tentative pulsations backwards and forwards. In old days the idea of magic attached itself to the exercise. At a time when the ability to investigate the angles of the pentagram represented a high degree of mathematical skill, we may imagine that some enthusiast, in a fit of that tender fun which is characteristic of scientific genius, conceived the idea of tracing the figure on his threshold, saying to his pupils, ‘No lying spirit can enter, nor can science degenerate into sorcery, if study is put under the safe guardianship of accurate mathematics.’ ‘Very slight inaccuracies,’ he would add, ‘leave room for the entrance of any kind of treachery and deception.’

It would conduce to the development of the imagination for intellectual purposes if children were made practically acquainted with the use of parallax as a revealing agent. This can readily be done by drawing a triangle of, say, an inch in height, and a small portion of a similar one with a base, say, three times as long as the base of the first. From the end points of this longer base draw the sides to the height of, say, half an inch ; and, in some simple way, call the attention of the child to the fact that, though all he can *see* of the large triangle reaches only to half the height of the small one, yet he can, and does, know that its apex is three times as far from the base as that of the other ; because, though there is nothing in either side to show how high it is meant to be, the parallax of the two fixes the point of junction.

Much discussion has taken place of late years about the choice of a book from which to teach Geometry. There can be no better textbook for the teacher to use than Euclid, on three conditions : That the child has been adequately prepared before the age for any formal teaching ; that the Euclid is not given to the child to learn from till he has gone through the Geometry of the important statements in the first two books without being distracted by the Logic ; and that each such statement is

put to him as not a statement but a question. E.g.: What is a circle? The answer to be elicited is: 'The line which I draw when my compasses are properly screwed up, so that the pencil-point keeps the same distance from the steel point all round, is a circle.'

What is a straight line? 'When I have drawn my stitch up as tight as I can without bending the card it is straight. Then the card is flat. If I draw the thread tighter the thread will still be straight, but the card will be bent or curved. As long as my stitch is slack, so that there is more thread on the front of my card than just enough to reach from hole to hole, the thread is not straight.' It is here that we perceive the advantage of letting a child associate his first conceptions of plane geometry with lines drawn, not with pencil, but with needle and thread.

Give the first axiom thus: Here is a strip of white cardboard; in the next room is a black strip; measure the white strip with this tape; cut the tape the length of the white strip. Now take the tape into the other room and measure it against the black strip. 'Is the black strip just the length of the tape?' 'Yes?' Now we will put the tape away. You cannot see the two strips of card at once, because they

are not in the same room. 'But is there anything you know about their likeness to each other?' 'They are the same length.' 'How do you know that, when you cannot see both at once?' 'Because they are both equal to the tape.'

Each axiom so arrived at is to be registered in the formula-book. The principal definitions and axioms of the First Book of Euclid are thus made out by the child and entered in his formula-book as his own discoveries. Then he begins on the propositions.

'Do you think that you could, with no tool but your compasses, draw a triangle with sides all equal? Let us try.'

Or, in the case of the theorems: 'Here is a triangle ABC ; can we make another triangle ABD , so that $AD = AC$ and $BD = BC$?' (After much experimenting) 'Yes, but D will have to be on the other side of the line AB from where C is'; and so on. We go on thus till we have registered Prop. XLVI.

All this time there should have been no attempt to quicken the child's perceptions by any magnetic stimulation from the teacher, whose personality and influence should be kept as much as possible in the background. The child has gone on unfolding his direct geometric perception at its normal pace. At Prop. XLVII,

it becomes even more important that the lesson should be as little as possible connected with instruction from a human teacher. It is, or should be, one of the most momentous crises of the mental life.

The question is posed thus: 'You know that any two sides of a triangle are greater than the third side. How much greater depends, you know, on the angle between the two. Now suppose we leave out of reckoning all triangles except those which have a right angle. Is there any way in which we could know the length of the hypotenuse by knowing the lengths of the other sides?'

The child is left to grope and fumble over that question for a short time every day—for perhaps a week; so as to get the sense of impossibility, of helplessness, well worked into his consciousness; because it is important that he should get an idea of the true relation of mathematical method to the sense of *helplessness* generated by the limitation of direct human faculty. You then tell him to make a right angle by placing three of his square counters or blocks in a row for one side and four for the other. He is then to make the hypotenuse, and see how many blocks go to make that. He will find that he needs five. He will then perhaps suspect that the law of the relation

is contained in some idea of sequence of numbers. Let him then try four, five, and six ; he will find that six does not make the hypotenuse long enough. After trying various measurements for some days, you again tell him to make the triangle whose sides are three and four and the hypotenuse five. Tell him then to build up the complete square on each side. Then tell him to count the blocks in each of his squares. Then tell him to add together the numbers of blocks in the two small squares. And then, if you can, have the tact and wisdom to be silent and let him think. Say no more of geometry that day. Next day let him build the hollow triangle and solid squares of 3, 4, 5 again. Then other right-angled triangles, such as those with sides 5, 12, 13 ; 8, 15, 17 ; &c. Use no unnecessary words.

When he is quite familiar with the process, tell him that some great mathematicians think that if ever we learn to talk to people in other planets the first letter of the interstellar alphabet will be the three squares done on a gigantic scale. The impression to be made on his mind is that one of the most sacred and solemn acts of a human life is that of coming into contact with the three squares on the sides of a right-angled triangle. If this

lesson has been reverently carried out, the pupil will be henceforth a true initiate of the order of Free-Geometers. There will be a similar lesson with regard to Algebra, to be described presently. We will add only a few words more about the Geometry.

You may now run rapidly through Book II of Euclid, still putting the propositions as questions, and working each out, first empirically with squared paper, next algebraically. When the child has completed Book II, he will be ready for the Logic of the subject. Tell him that Euclid arranged various geometric propositions in such a way as to show, not that any one of them is true, but that they are all connected together. Let him then begin to learn Euclid from the book itself, in the old-fashioned way. Insist now on the utmost rigidity in following the exact steps of Euclid's reasoning.

The cultivation of the mathematical imagination should include not only its development but its orderly and systematic exercise. A child is too often made to pass from a particular case which suggests a law to other particular cases which require an application of it, without a sufficient amount of intermediate drill in analysis of the law and in tracing out its results exhaustively. Between the time when a child

handles an actual cube, cuts sections, &c., and the time when he comes, among his ordinary geometrical exercises, to problems requiring him to draw the elevation of a cube cut in some particular way, there is a period when he finds it useful, and very delightful, to go through a set of processes in imagination and to express them in his own words. 'I think of a cube. I think of slicing it, beginning at one angle. I get a point, then a little triangle, then bigger triangles. The corners are cut off, making a hexagon with unequal sides. The odd sides increase; the hexagon now has all its sides equal. The sides which were longer before are now shorter than the others. They disappear and the hexagon has become a triangle again, but with its angles in the other directions. The triangle becomes smaller and smaller. It becomes a point. It disappears.'

A little systematic drill of this sort makes subsequent drawing out of complicated problems much easier, for the child is accustomed to seeing, with his mind's eye, coherent and orderly modulations of form.

We prepared for Algebra by means of the closed boxes of counters. Practice has been given in two or three scales of notation; on one day each box may contain (or each red counter stand for) eight white ones; on another day

twelve ; and so on. This preparation forestalls many of the difficulties which might otherwise obstruct progress up to, and not including, Quadratic Equations.

‘Do you mean to tell me,’ said James Hinton, ‘that there are men on earth wicked enough to *tell* a boy *how to do* a quadratic equation?’ I would rather ask : ‘Is there any country on earth, calling itself civilized, where children *need to be told* how to do quadratic equations?’ Long before the child will need to do quadratics in the school course he should know all about ‘how,’ without any telling, by a process of initiation. Among his ordinary simple equations should be set one of this kind :—

$$x^2 = 9.$$

He will readily guess the positive answer. Ask him whether any other value of x would make $x^2 = 9$. Give him till next day or next week to find out. When that difficulty is cleared up, set :—

$$x^2 + 10x = 24.$$

He will probably be baffled. Let him try for a few minutes daily till the sense of *baffle-*
ment is complete. Then give

$$x + 5 = 7.$$

Bid him square both sides. Then show him his own answer side by side with the impossible equation. Make him examine in detail the

steps of the multiplication which led to the originating of the coefficient 10 and the numerical term 24. Then let him find out for himself 'how to do' his next quadratic; which, of course, should be on the same easy scale.

An incident in my own very irregular course of study may be suggestive to some teachers. At sixteen I was given a book on the Differential Calculus with the statement 'No one can understand why this method should answer; it seems contrary to the nature of things; but if we trust to it the results come out right; it is like putting corn into a mill-hopper; you cannot see what happens, but it comes out flour at the end.' I was far from satisfied, either with this account of the science or with the reasoning in the book. I soon found in the library an old book of Fluxions, into which I plunged with delight; it was perfectly intelligible to me. After I had been revelling in my prize for a week, my father found me with the book and took it away, telling me that the Fluxion notation was old-fashioned and inconvenient, and quite given up now at Cambridge. I went back to my differential book, and found, to my great delight, that it was now perfectly clear to me. Two years later I met George Boole. When he heard my story he told me that the differential notation is indeed more

convenient for practical purposes than the fluxional ; but it is less clear as to the philosophy of procedure ; and that no young person should see a differential book till he has made some study of Fluxions. My own conviction is that I was partly indebted for the ready response of my mind to the Fluxion-method to the fact that at an early age, when my fingertips were still sensitive, I had the habit of playing with a bandalore. The thrill of the fingers at the upper and lower dead-points lodged in the brain a comprehension of maxima and minima worth more than all the explanations that ever were written or spoken. The bandalore-pause is a phenomenon similar to the stationary length of days and nights at midsummer and midwinter. It might be well to lead a child to observe this.

There are several other precautions which might be taken, with a view to forestall difficulties of imagination in connexion with the Calculus ; one of these is to set the question : How many terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, &c. must we add together to make up the number 2 ? Having got the pupil to realize that no number of such terms would quite amount to 2, but that every fresh term brings the sum a step nearer to 2, tell him that 2 is the *limit* of the series $1 + \frac{1}{2} + \frac{1}{4}$, &c. Let him enter this

statement in his formula-book. At about the same period he should be shown that, though neither a circle nor a straight line is really an ellipse, yet a straight line is one limit of a series of ellipses and a circle is the other. Get him to understand clearly the difference between a finite straight line which is the limit of a series of ellipses (or, in other words, the path of a point pulsating between two fixed points) and one which is cut from a longer straight line; that a bit cut from a stick is of the latter kind, whereas a seedling shoot in the process of being generated by the flow of the sap is of the former; say also that no living line is ever more than approximately straight.

Another kind of lesson on limits can be given in connexion with such a book as Hinton's *Life in Nature*, or the Chapter on Skeletons in his *Thoughts on Health*. Show that chest-nuts, each potentially round, press each other flat inside the green outer case. Show the round cell of a solitary bee, made to suit its own shape; then show how, when some bees took to living together in holes, or tree-trunks, and had to economize room, the pressure on space straightened out the sides of the cells, till the limiting form, the hexagon, was reached. Show a similar process going on in round sheaves of straw piled together under pressure.

All these ideas, suggested at suitable ages, prepare the mind to receive the Higher Mathematics as vital air or natural food.

By the age of fourteen an intelligent child is quite ready for a lesson in preparation for the doctrines of Singular Solutions and of Hyperspace. The following is interesting to eager young minds, and clears up the conceptions of ill-trained adults.

Get a stem, at the top of which grows an erect flower, by preference monopetalous and tubular, with a pointed bud growing beside it, the tip of which does not reach as high as the top of the flower. (There are several *Gentians* and *Campanulas* which answer the purpose.) Say to the pupil: 'Imagine a very thin horizontal section of this flower, cut above the height of the tip of the bud, and pasted down on paper. Imagine a little child, who has been accustomed to draw circles with coloured pencils in compasses, looking at the paper. He might suppose the flower-section was made in some such way as he makes his coloured circles; that it was made originally just as he sees it, without anything more than he sees. Now imagine a section cut a little lower, so as to shave off the point of the bud; imagine the whole of this new section, bud-tip and all, pasted on paper while the child is not

by and afterwards shown to him. He would suppose that the circle was drawn with compasses, and the dot made separately at random. If you told him that the whole came into being as one thing, that the point was part of the same growth as the circle, he would be quite puzzled ; he would not be able to conceive of any process for making the circle which would involve a point at a distance from it. But if you explained to him that the circle did not come into being as a flat circle at all, but was really part of a thing which had height as well as length and breadth ; that the sap was going up a stalk till something stopped it flowing up straight, and then the stalk's growth stopped, and it split in two, and one side grew into a tubular flower and the other into a bud ; then the child would understand.' 'You remember that we could not understand the relation between the hypotenuse of a right-angled triangle and the lengths of the other two sides till we thought of the three lines as sides of a two-dimensional figure. Now you see how the relation between marks on a flat surface may be unintelligible till they are treated as sections of a three-dimensional figure.'

Soon after such a lesson as the above it might be well to let the pupil read Babbage's Ninth Bridgewater Treatise ; not as

a lesson on Mathematics ; but as a sort of Sunday or Holy-day book. It is unfortunately out of print—chiefly, it seems to me, because of the preference of the average reader for vague guesses and insincere sentiment over exactitude in matters of mental and moral development. It will be reprinted, if England comes to its senses before our civilization crumbles. If you can procure a second-hand copy give it to the pupil at fourteen. Henceforth let him learn science from cut-and-dry textbooks ; as much of the poetry of science as he is capable of feeling will occur to him spontaneously as he goes along.

CHAPTER V

ETHICAL AND LOGICAL PREPARATION

IN the good old days, when Classics and Theology formed the staple of a liberal education, the science of a good school might be represented by a 'writing and ciphering-master.' This person's rank was much that of some Army drill sergeant who came periodically; he had no status on the staff of the school, and little influence over the boys; and of course none at all on the homes whence the boys came. The classical masters, usually clergymen, set the tone of everything and struck the key-note of all the relation between children and their elders. But in these days the science teachers, the mathematician, the electrician, the botanist, are a real influence; the child's relations with these persons go far towards setting the tone of his relations with adults generally. Accustomed as he is to the free kind of intercourse which prevails between teacher and pupil in a laboratory or on botanical excursions, he does

not quite get back, even during Latin class, into the attitude in which our grandfathers, as boys, stood to the autocratic classicist. Whether for good or for ill, the omniscient Dominie is fast disappearing from our civilization; indeed, he would already find himself out of place in any good school.

In the old days a certain ideal of nursery discipline evolved itself, suited to prepare the child for entering into relations with the classico-theological schoolmaster under whom he would presently be placed. That ideal was summed up in the old rime:—

‘Speak when you’re spoken to, Do what you’re bid,
Shut the door after you, And you’ll never be chid.’

‘Speak when you’re spoken to,’ in this old summary of the whole duty of the child, did not merely mean respond when you *are* spoken to; it distinctly was understood as an injunction not to speak till spoken to; never to start questions; never to ask for anything till it was offered you. It was in line with nursery proverbs, such as: ‘Children should be seen and not heard;’ ‘ask no questions and you’ll be told no lies,’ and so on. About that ideal nothing need be said now, except that it is inadequate to prepare children for entering into right relations with science teachers.

Science teachers found that the old sort of

nursery discipline, what I may call the speak-when-you're-spoken-to training, sent up children to their classes unable to enter thoroughly into the relation which they wished to establish, which is found to afford the best condition for imparting and imbibing a conception of scientific method. Therefore, where they have access to nurseries, they endeavour to start a new kind of elementary training, a discipline, comparatively lax indeed at some points where the old sort exerted great pressure, but also stringent at some points where the old sort exerted no pressure at all.

I am not going to enter into a discussion as to which of the two methods is absolutely best. Personally, I incline to think that, at our present stage of evolution, the very young may still need a certain amount of the old sort of nursery training; but the point to which I have to call attention is that, where we have relaxed it, and in so far as we have relaxed it, it has been because we wished to make room for another mode of infant discipline which suits our purpose better. If one no longer says to children: 'It is naughty to disobey; if you do, you will be slapped now and worse than slapped in the next world,' one does try to make them understand that it is stupid not to be prompt and accurate in carrying out instructions, and that,

unless they can become so, they will be useless and in the way, and not able to take a share in the work and life of the home.

It is painful to find that many persons are following our example in the matter of relaxing who have no idea what can be best put in the place of the old discipline, nor any special intention of putting anything in its place. Many parents seem to think there is something advanced and progressive and twentieth-centuryish in the mere fact of relaxing the old speak-when-you-are-spoken-to code of infant ethics. This is a great mistake; *mere* relaxing without compensation is never progressive; it is a relapsing back towards animalism; it is de-culturing; and it is more de-culturing in proportion to the wealth of the parents—that is to say, to their power of emancipating their children from that pressure of crude, hard, physical struggle by which Nature evolved us upwards.

If Science is making children too wide-awake to think there is any merit in believing what they are told and doing as they are bid, as it most assuredly is doing, Science is bound to set about helping parents to organize such discipline as shall prepare children to judge what is worth believing, and to obey their own higher selves: two things which the undisciplined and careless never can do. I feel that

we science teachers will be guilty of grave neglect if we allow any slipping between two stools to take sanction from our supposed example. It may be well to give parents a picture of the kind of ethical code which some science teachers are trying to introduce into the nurseries in which they have influence. Perhaps I may be allowed to sum up that code in a new nursery rhyme. (It is doggerel; but so was the old one.)

‘Know what you mean to say; think what
was said;

Fence round your fairyland; then you’ll have
a clear head.’

What I may call the know-what-you-mean code of nursery ethics includes these elements:—

(1) A training in the sense of personal responsibility for detail.

(2) A training in giving clear reports of what has been done.

(3) A training in knowing when an incident is closed, when a cycle is completed.

(4) A training in answering the exact question that was asked.

(5) A training in distinguishing apart different sources of knowledge.

(6) A training in keeping the world of imagination and of hypothesis distinct from the world of ascertained fact.

(7) A little, though very little, elementary training in judging what is most relevant to the matter in hand.

All this can be done, and is being done, before the school age, without intellectual strain to the child, and without the aid of special educational books or apparatus, by means of the water-tap and the wash-hand towel, of pinafores and spoons.

There comes a stage in every child's life when he is anxious to be sent messages; and this phase can be taken advantage of to train him in one or two habits which it is difficult to acquire at a later age, and the lack of which hampers the development of the scientific faculty. When a child is two or three years old, you ask him : ' Would baby like to go a message for mother ? ' When you find him willing, you say : ' Put down that toy (or whatever he may have in his hand) and come and stand in front of me : put your hands straight down, head up, look me straight in the face, say : " Please, Anne, a spoon. " ' Say it again. I am going to send you to Anne to fetch a spoon. What are you going to say to Anne ? Now you will say nothing else, don't talk, don't play on the way for fear you should forget. Now tell me once more, what are you going to say to Anne ? ' When the child comes back with the spoon, you say to him : ' Now go back and say : " Thank you, Anne. " ' What are you going to say

to Anne? Well, now go and say it.' When he comes back the second time, you ask him what he said to Anne. If he cannot remember, or is not clear whether he said it properly, you send him back to try again. As soon as he brings a clear and crisp report of having given his message properly, you at once restore whatever he may have had in his hands before you began. This habit of withdrawing all possible sources of distraction before business begins, and restoring whatever you deprived him of directly the business is completed, is of importance; it answers the same purpose as is fulfilled later by making a child put a big A opposite the final answer to a sum. All these precautions help to induce the habit of knowing when a cycle is completed, a duty fulfilled, an incident closed.

Next day the message may be, 'Please, Father, a pencil,' or 'Please, nurse, a clean pinafore,' but it is well, while varying the material of the lesson, to keep the routine exactly until it becomes quite easy and mechanical; until the mere fact of being called to a lesson at once throws the child bodily and mentally into the attitude of 'stand at attention.' After that you tell the child that whenever you send him to fetch anything he may say 'thank you' to the person who gives it to him before bringing it to you; but he is still not to talk of anything

else when on his way. Just at first you will have to explain to the household that they are not to tempt the child to dawdle or talk when sent on a message ; but as soon as he is old enough you may tell him that if any one speaks to him when he is on his way, he should say : ' I am on a message for Mother, I will come back to you when I have done what she told me.'

As early as possible you choose some little function, which the child has learnt to perform (such as washing his own hands), as a means of training him further into the sense of responsibility in carrying out orders. For instance, when you see that he is able to wash his hands properly you explain to him that it is not safe for him to touch the hot-water tap, as the water is sometimes hot enough to scald him. You tell him that he is not to touch the hot tap unless you are there to give him leave. If you intend that any other person shall have authority to give leave, mention that person formally at once ; say : ' Unless nurse or I give you leave,' or ' Unless Father or I give leave ;' and, having said so, let it be understood that any grown-up person whom you have not formally mentioned may draw hot water for the child if she wishes, but may not give leave for him to touch the hot tap. At this point there will probably come little difficulties with servants

and relatives. 'As if I didn't know as well as his mother when the water is too hot!' You must explain to the complainer that, if the business in hand were *only* preventing Jacky from scalding himself, you would have perfect confidence in her care; but that Jacky is just now getting a lesson about what he is responsible for, and to whom, and that no confusion should be introduced into his mind. A little tact and a little firmness are needed to soothe affectionate jealousies; but the results of the method are so satisfactory to the whole household that people soon begin to say that, after all, the mother seems to have known what she was about. Well, you send Master Jacky up to wash his hands, giving him minute directions as to his order of procedure, which you make him repeat each day until you find that he no longer needs to be reminded of them. Every day he comes back with his report, which may be as follows: 'Cook was upstairs; I asked her to draw me some hot water; I washed my hands; I used soap; I rinsed the soap off; I used my own towel; I think I wiped my hands quite dry; I did not touch any one else's towel; I put my own towel back on the proper rung;' varied occasionally by: 'There was no one upstairs to draw hot water, so I used cold and gave my hands an extra scrubbing; I think they are quite clean,' &c.

The report must include the details as to which you cautioned him. Amongst its other advantages this has a tendency to check a child's natural inclination to occupy the conversation with details of his own affairs and performances. He understands that reporting the details of what he has been about is a piece of business to be done at a certain hour of the day, and heard no more of.

Jacky may come down brimming over with information about something he has seen while upstairs, some effect of light from the falling water, or a big bubble that the soapy water made 'with all sorts of colour in it.' We must be very careful not to discourage this eager and glowing interest in natural phenomena; but if we can, without discouraging him, introduce the conception of graduated relevance, it will be so much the better for his future clearness. This may be done by asking him whether he thinks he could manage to tell you all that over again in a different order, to tell you the business part first and about the soap bubble afterwards.

Care must be taken that all this accuracy does not degenerate into superstition about particular actions, that the child attaches chief importance, not to this or that action, but to accuracy in carrying out whatever are the orders for the time being. We need to induce the

algebraic, as against the superstitious, conscience. In algebra, a must stand for exactly the same value throughout the one problem, but may stand for something quite different in the next problem. Great use may be made in this respect of a visit to a strange house. You may say to a child, for instance : ' I am going to send you upstairs (to fetch me a handkerchief, &c.). This house is not arranged for children ; it is different from our house. The wall-paper is white, you might easily stain it ; the stairs are slippery ; the windows have no bars in front of them. In this house you must hold tight by the banisters, come downstairs very slowly, never touch the wall-paper, and never go near a window that is open at the bottom. Now tell me what are the things you are not to do in this house ? ' (Touch the wall-paper and go near windows which are open at the bottom.) ' And what are the things which you are to be careful to do in this house ? ' (Hold by the banisters and come downstairs slowly.) When he comes back he brings his report formally : ' I fetched your handkerchief ; I held tight by the banisters ; I came downstairs slowly ; I did not touch the wall-paper ; there was a window open at the bottom, I didn't go near it ; ' varied by ' there were no windows open at the bottom to-day.'

It must be particularly noticed that a child can be trusted about amongst dangers, such as hot-water taps and open windows, *immediately* after such a catechism and while he has it on his mind that he will have a report to give, at a much earlier age than it would be possible to trust him vaguely and promiscuously. Once, or at most twice, in the day is as often as a child under six ought to have the strain of responsibility thrown on him; for the rest of the time he should be allowed to be his natural, careless, impish self; and other people, not himself, should guard him from mischief and danger.

If a child is sent a lesson-message, and is asked on returning whether he gave it, he will sometimes answer, 'I could not find cook,' or 'She was not in the kitchen.' For the mother's own immediate practical purpose that is sufficient; but logically speaking it is not a categorical answer. This elliptical mode of speech is allowable in ordinary conversation, but I am now describing specific lessons intended to evoke the algebraic super-consciousness. They are intended to serve the same purpose as is served later by occasionally making a child write out in full every step in the working of a sum, including many steps which, on ordinary occasions, he leaves out.

With a view to preparing for scientific train-

ing it is well to accustom children early to answer the exact question asked before entering on any side issue. The question 'Did you do so-and-so?' must be answered either as 'yes' or 'no.' 'She was not in the kitchen' is the answer to a further question; and if said at all must come after the answer to the one actually asked. This habit of logical distinction is of great use in preventing friction with the future science teacher and confusion to the pupil. At an early age, when no lessons are on hand, when learning to make the distinction is itself the lesson, it can be made non-worrying and even interesting to the child. One does not wish children made to speak, habitually, in a formal or priggish way; but it does seem desirable that, before a child goes to school, he should be in such training that the request, 'Please answer the question I asked' shall meet with intelligent response, shall not produce in his mind a sense of mere bewildered pain.

But I must here repeat the caution given in Chapter I. The habit of alternation of attitude is *the* important one to set up with a view to a scientific comprehension of things. The child who is getting lessons in the algebra of ethics by the method recommended in this chapter will be more likely to turn out really scientific, if his conduct at ordinary times is directed by

an ordinary well-mannered mother or a good old-fashioned nurse, than he will if constantly in the hands of any would-be scientific faddist.

The power to distinguish apart different sources of knowledge may be evoked by an occasional very gentle, playful sort of 'how do you know?' jesting. The lesson might well take place when a child has poured out a heterogeneous mass of scraps of information gathered during his walk. 'Oh! Father! there was a red butterfly with black spots on his wings. And he had a trunk just like an elephant's. And he drinks honey with it. And that butterfly and the bees were so hungry; there were a lot of bees. And there's a butterfly with a '*normous* trunk—so long—and he ran a race with a great white flower,' &c. 'How do you know there's a red butterfly?' 'I *saw* him; he flew across the road in front of me.' 'Could you see the black spots as he flew?' 'No; but he went and sat on a flower in the hedge; and then I went close to him and saw the spots.' 'So he had a trunk just like an elephant's, had he?' 'Yes; just like an elephant's.' 'Are you sure it was quite as big?' Then, 'So the bees were very hungry? Did they say so?' 'No, they couldn't speak.' 'How do you know a person is hungry when he does not say so?' 'Well, they were all in such a bustle; they went

so quick from one flower to another.' 'Well, I'm in a bustle sometimes just after breakfast ; but I'm not hungry then.' 'You are going off to your office ; bees haven't an office to go to. They were getting honey.' 'How do you know it was honey?' 'I sucked one of the flowers myself ; and it was sweet ; and Mother said the bees were getting honey.' 'Do you put things in your mouth when you are out walking?' 'No ; not unless Mother is there and gives me leave ; but she said I might suck one of those flowers.' 'That's all right. Well, the bees were getting stuff that tastes sweet, and Mother told you it was honey. And you think they were hungry because they worked hard ; I work hard often when I am not hungry.' 'But Mother says you work to get money ; bees don't get money.' 'No ; they don't get our sort of money ; but are you sure there is no one at home they want to get something else for?' And so on. About the butterfly with the eleven-inch trunk you elicit the avowal that no one told the child about it, and he did not see it ; he heard Mother telling Auntie about it ; she had read about it in a book. By asking whether the big white flower in Mother's book had legs to run with, you make it evident that the child does not understand what is meant by an evolution-race ; and if he answers as a puzzled

child sometimes will, 'Well, I heard Mother telling Auntie about it, so it must be true,' you can explain that Auntie knew what Mother meant, but he apparently does not ; so he may as well leave that part of the tale out till he is older, and for the present only tell people about things he really understands. Of course one must keep a discreet limit on the extent to which one stirs up a child by this kind of 'how do you know' questioning ; specially one must be careful that it does not degenerate into becoming a worry and a check on his flow of talk. But a little of it, occasionally, is of great value in preparing the way for science teachers by getting the child into the right attitude to receive their instruction.

It should be noticed that a child accustomed to ever so little of it is likely to ask his elders how they know so-and-so. Parents should be prepared for this. If the child says : 'How do you know ?' in a saucy or ill-tempered tone, it is rebuke enough to say : 'If you really want to know, you will ask me nicely.' If the question is put politely and seriously by a child, the parent should be prepared to answer honestly either 'I saw it,' or 'I read it in a book which I believe to be true,' or else 'I cannot explain yet how I know ; I know in a way which you cannot understand till you are older.' Parents

who object to the 'how do you know' line of inquiry being started at all in their nurseries ought to reflect that the passively receptive attitude towards elders, though a satisfactory preliminary for a merely classico-theological education, puts needless and unfair obstacles in the way of science teachers.

It is a good thing to afford a child the opportunity for perceiving that there are three ways in which he might become aware of the connexion between cause and effect, e. g. in which he might learn which handle sets a certain bell ringing. In one case Mother might have told him; in another he may have heard that particular bell ring when that handle is pulled so often as to feel compelled to connect the two; in the third case he may have followed, once and for all, a series of wires or cranks all the way from handle to bell¹. All these ways of learning are equally legitimate, and all are useful; but he should be clear which he is relying on in any given case.

A child is very prone to mistake the intensity of his own conviction that a statement is true

¹ The three methods of acquiring instruction about the bell illustrate respectively (1) learning from authority, by inheriting the stored-up knowledge of the race, (2) ordinary scientific induction, such as that by which a law of nature is inferred, (3) the specifically mathematical induction. Boole, *Laws of Thought*, p. 4.

for proof of its truth, or even for comprehension of the statement itself. It is astonishing to see with what simple conviction a lad or girl will sometimes assure one that he or she *quite understands* a certain process in Science, say the process for finding G. C. Measure, or for solving Quadratic Equations, or for doing something with chemicals, as to which one finds that in reality he only knows empirically the steps which he ought to take and the order in which to take them; he has been shown what to do; and repeated trials have proved that he can do it aright. This confusion points to something unscientific in the nursery training. A child who was scientifically trained in infancy will sometimes say, 'I know how to do these sums; I do them easily, and I always get them right; but I don't understand *why* they come right.' Now that kind of pupil is the delight of a good science teacher, and a perplexity to a bad one. The more logically we treat our children in infancy, the more likely they are to make the neighbourhood uncomfortable for unscientific teachers of science.

True, it is often necessary to know what to do and how to do it, when we have no means of knowing why it answers. But it is very unscientific not to be clear in one's own mind when one only knows that a process does

answer, and when one can see beforehand that it must answer. Nothing can be thoroughly done in mathematics till the child quite understands the difference between these two things; and it is unfair that a mathematical teacher should be interrupted by the necessity for insisting on the difference. Very many ordinary teachers are themselves so unscientific that they do not know how to explain it; all the more is it desirable that their pupils should be clear about it before entering their classes. Indeed, it is not a thing that can be made clear by mere explanation; it is a matter of mental habit, and must be begun early.

The same kind of reason holds good for accustoming children occasionally to that kind of mental action which is involved in answering such apparently foolish questions as: 'Are you sure the butterfly's trunk was exactly like the elephant's? Was it quite as big?' A very great deal of the difficulty of teaching mathematics would be non-existent if the parents would accustom children very early, by means of jokes, to realize that a thing may be like another thing in one respect and quite unlike in another.

There is nothing more important in teaching Algebra than to make the pupil understand clearly the difference between an equation of identity (a dozen is twelve), an equation of

real values (e. g. the circumference of a circle is three and a fraction times the length of the diameter), and an equation of hypothesis or arbitrary equation (such as : Let a red counter be supposed to stand for ten white ones). It is impossible to teach Algebra properly unless this distinction is quite clear ; and it is almost impossible to make it so during the Algebra lesson itself, where the material in which we work is abstract. I cannot say that I ever found a child to whom I failed to make the distinction clear in Arithmetic, where the values are concrete, if I took trouble enough ; but I have met many girls of sixteen to whom it seemed a new and puzzling conception ; and I do think it is disrespectful to the teacher, and unfair to the better prepared pupils, to have to stop the work of a class in order to teach to the others something which ought to have become an easy and automatic action of the mind long before.

Again, such words as *same* are very misleading if used without care to distinguish their various meanings apart. If you say, ' This blue aster is of the same species as the pink one I had last year,' or ' My dog is of the same breed as my neighbour's,' that connotes that the two asters or dogs are (in spite of superficial difference of colour, &c.) so nearly alike as to

need similar food and care ; experience gained in treating the one will be of use in guiding us how to treat the other. But if you say, ' This gnat is the same which was swimming as a wiggler in my aquarium this morning,' that does not imply anything of the sort. The wiggler of the morning would have died if kept out of water : the gnat of the afternoon will die if kept in it. You can infer nothing about the one from what you know of the other.

All this may sound very childish, and so indeed it is. This is the very reason why I venture to think it is well when this part of the work of training for Science is done, and well done, at a childish age, pleasantly and as mere fun, instead of throwing upon the teacher of science classes the burden of introducing such elementary conceptions for the first time to young persons whom he is preparing for stiff examinations in Science.

The effect on a science class is as distracting as it would be if a literature teacher had to interrupt a lecture on Shakespeare to help some of the pupils to spell out ordinary English words.

We now come to the fencing round of the imaginative faculty. As we can never be sure how far a child's statement is consciously and voluntarily false with intent to deceive, many

people prefer never to accuse a child of lying. Children often have no clear notion of the boundary line between fact and imagination. And indeed the modern child is far too logical to accept from grown-ups who tell fairy tales any such dictum as that it is wicked to say what is not true. But because we no longer talk to small children about the wickedness of lying, that is no reason why we should not try to help them to learn the art of telling truth. For it is an art; at least as difficult as playing the piano; as little to be done at any given moment by mere effort of will; as necessary to begin learning young. Most children have access to some sort of fairyland in which they spend a good portion of their existence. There is nothing unscientific in that; indeed, every scientific man worthy of the name, every one who is not a mere mechanical adapter of other people's discoveries, spends a good part of his time in that grown-up sort of fairyland, the world of scientific hypothesis. It is from this fairyland of his that he draws the inspirations which guide his researches. We have each a private domain of our own in that Unseen World; and no mortal has the right to deny the truth of what another says he sees there.

I received a good lesson once from a little friend of mine whom I found on the sea shore,

earnestly imploring some fishermen to bring her home a mermaid. I told her that there are no real mermaids. 'Oh yes, there are,' said the little maiden imperturbably. As I persisted, she became troubled; but presently recovered herself and said cheerfully, 'Oh well, if they are not in this world they must be in fairyland, for I *know* there are mermaids somewhere.' I had received a well-merited rebuke. What right had I to say there are no real mermaids? what do I know about it? There may be some in Mars for all that I can tell to the contrary; and the child may have had some *Ahnung* of something which I do not understand. What I did know, and what I ought to have said, was that there are no mermaids living under such conditions that fishermen in this world can bring them in boats to little girls.

Many children have imaginary friends, and give names to those friends; besides having a name of their own for the imaginary world in which these friends are supposed to live. This simplifies the work of teaching the child to be truthful. One can listen gravely to any sort of statement he pleases to make, and then ask quietly, 'Did all that happen in this world or in mermaid world?' (or fairyland, or dream-world, or Charley's world, or whatever name he gives to his bit of the Unseen). He may be

puzzled at first ; he really does not know which world he has been looking into. But he gradually learns how to distinguish between the visible and the invisible worlds ; to know an impression, however vivid, on the mental retina from the image projected from the outside on the physical retina. Sometimes he will confess that part of the story happened in the outer world and part in fairyland. This mixing up of things should be discouraged, in the same spirit in which we discourage writing down tens in the units column of his little sums ; we do not assume any intention to mislead ; we assume that he would have preferred to locate the things in their proper categories, but was not expert enough, or was too careless, to do so properly, and must try again.

If we suspect that the child is giving a garbled version of some transaction to screen himself from blame, it is well, before asking any other person concerned what were the facts, to ask the child himself what version *he thinks* that other person would give, e.g. When he says, 'I didn't break the plate ; I fell up against the table and the plate fell and broke itself,' if you ask, 'What do you think nurse will tell me about it?' the child will perhaps answer, 'I think nurse will tell you that she had told me not to go near that table at all while the crockery was

on it.' A child who has thus corrected his own one-sided statement has had a very good lesson, and been much helped to become clear-headed and truthful. Nearly all the scientific truth known to man has been arrived at by successive processes of correcting error ; the condition for telling truth is not the mere negative fact of abstaining from inventing falsehoods, but the acquiring of positive skill in distinguishing our own inventions from fact before we speak to men. Now the child's parents should be to him a sort of outer conscience ; he should not be reproved for giving to them fact mixed with fiction, but helped to distinguish which is which, before either he or they take action in consequence of his statements or mention them to other people.

Suppose one suspects that a child is inclined to be really untruthful, one might sometimes take the opportunity, when he says he is hungry or complains of a little pain or hurt, to explain to him that there is no use in trying to bathe, or bandage, or otherwise minister to pains and aches and hungers in fairyland ; and that one cannot be sure which world the statements he makes refer to.

Few things that we do are more unscientific and more confusing to a child's logical faculty than telling him, about some of his pains, that

they are not real. Many so-called imaginary pains are due to inspiration, or the effect on the sensory nerves of over-rapid illumination from within. They are so real that they have been known to re-act on the fleshy tissues, producing actual swellings or haemorrhage from the skin. Imaginary pains are as real as anything in a child's experience. But the true remedy for them consists in a habit of exerting the will in turning the mind to external objects. The habit of seeking what we may broadly call medical remedies for pains which come to the flesh from the imagination is a dangerous one ; it is largely responsible for the alcohol and morphia manias ; it should be checked at its source in childhood. The way to check it is not to say, what is false (and, I venture to add, profane), that these pains projected from fairy-land are not real, but to accustom the child to acquire the power of controlling them from within instead of seeking help from without. The importance of this habit to health and morals is obvious enough ; my present purpose, however, is to call attention to its importance with regard to the child's future science teachers. The pupil who limply and helplessly flings himself on his teacher for help in difficulties which he ought to grapple with in his own mind and conquer for himself is very difficult

to teach properly ; while, on the other hand, many a mind possessed of really scientific insight has been wrecked by the bad habit of pondering for weeks or months, trying to set to rights, by sheer thinking, what should have been cleared up at once by appealing to a teacher or a book. And I am convinced that every step gained by a young child in the direction of clearly distinguishing the within from the without helps to lay a good foundation for all his future relations to Science.

Many objections have been made to some of the above suggestions ; but they seem to me to be the outcome of a dislike to cultivating the essentially algebraic super-consciousness in young children at all. As I said in the first chapter, it is not my business to decide whether this is to be done, but to suggest how those who wish to do it can best accomplish their end.

The physical attitudes, habits, and modes of handling things necessary in the chemical laboratory differ in some respects from those acquired in ordinary domestic and social life ; so also do the moral and spiritual habits and modes of handling facts necessary in an intellectual laboratory differ a little from those ordinarily considered amiable and agreeable. Those who wish to give their children a scientific training had better count the cost.

One word, however, I would venture to say to the objectors. Whether they like it or not, Science is sweeping on, and swallowing whatever stands in its way. Now no ordinary teaching of ethics constitutes a safe basis of conduct for the young person immersed in a scientific life, for that life is one of actual communion with the Unseen, the Eternal Organizer, who is also the Eternal Destroyer. It is a reflection of creative activity, an incessant process of reorganization of thought-material. Only those who have the orderly habit of organizing thought-chaos, of correcting the disorderly first impressions by reference to laws of organic thinking, can be morally or spiritually safe in that glorious whirlwind which we call scientific progress.

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